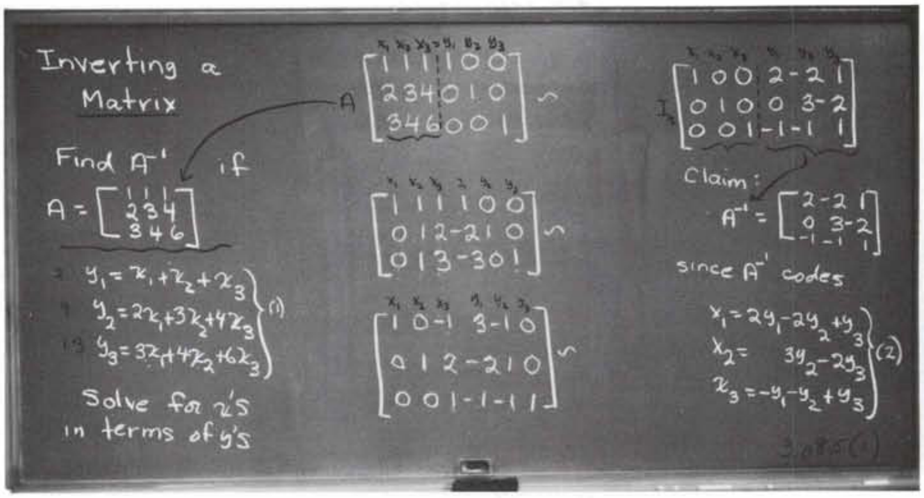
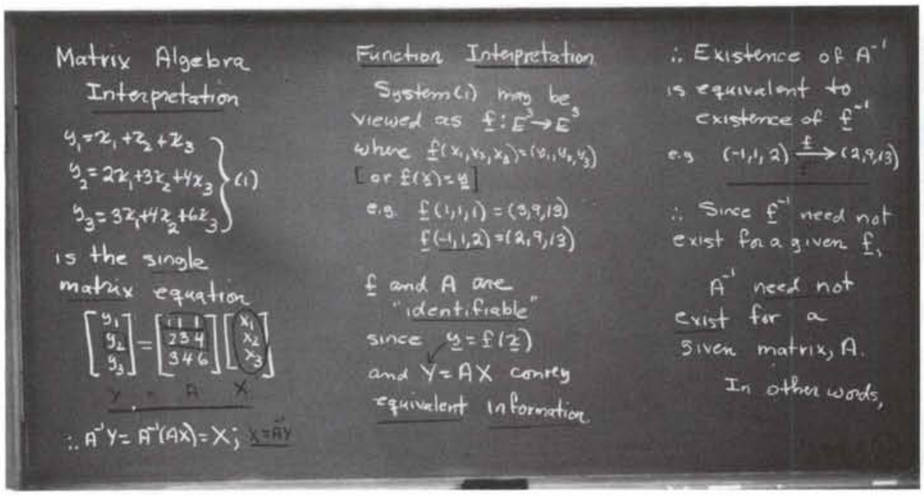
Unit 3: Inverse Matrices

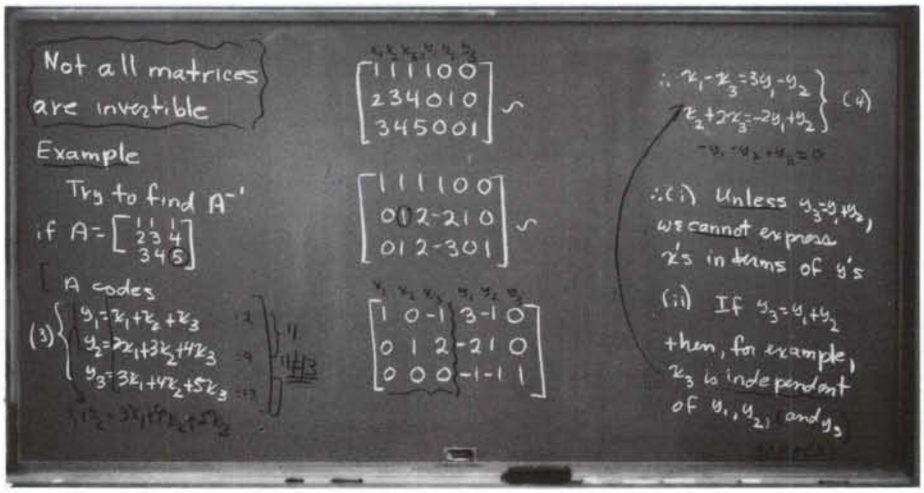
1. Lecture 4.030 (This lecture will also apply to Unit 4.)



a.

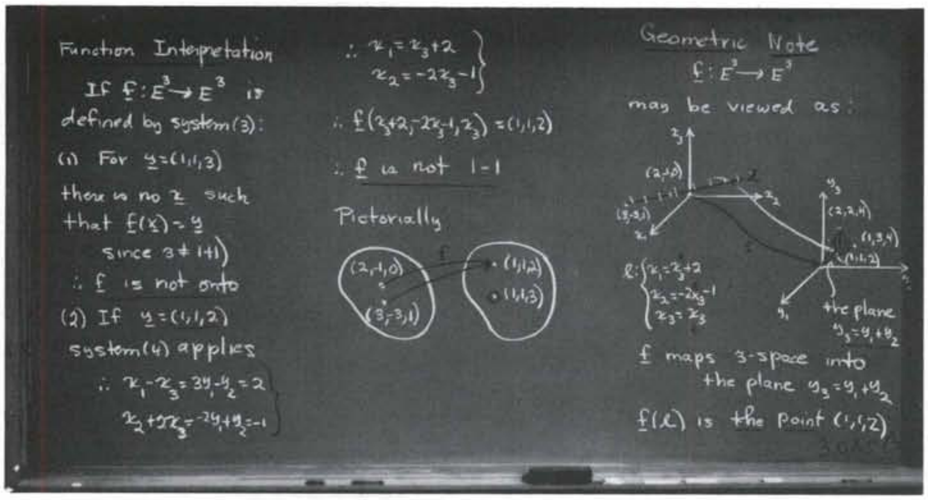


b.

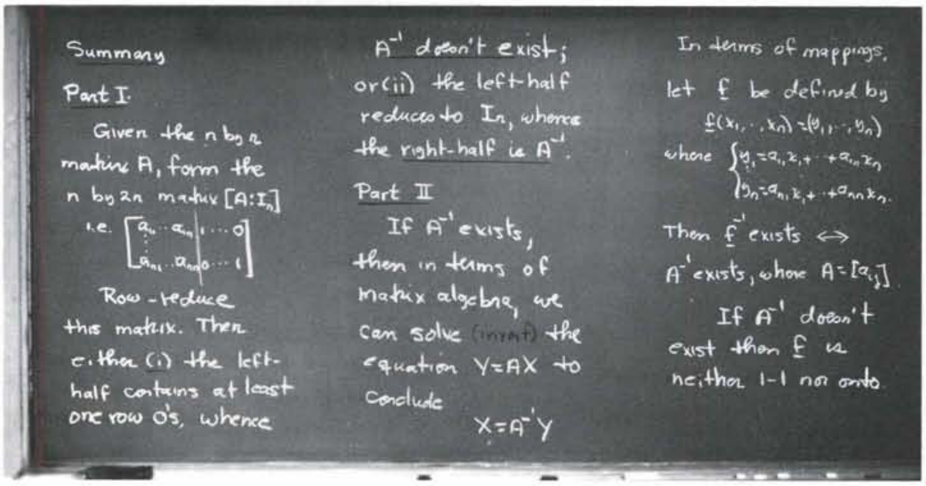


c.

Lecture 4.030 continued



d.



e.

2. Read Supplementary Notes, Chapter 6, Section E.

3. (Optional) Read Thomas, Section 13.3.

4. Exercises:

4.3.1

Let

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix},\ B = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix},\ C = \begin{pmatrix} 5 & 4 \\ 6 & 5 \end{pmatrix},\ 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Solve each of the following matrix equations.

a. $\frac{1}{3}X - AB = C$.

b. $AX - BC = 0$.

4.3.2

Let

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and let $X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$.

Assuming that $ad - bc \neq 0$, solve $AX = I$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

4.3.3

Compute A^{-1} if $A = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$, imposing any necessary conditions on a or d to insure the existence of A^{-1}. Generalize this result to find A^{-1} if

$$A = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix}$$

(i.e., $A = [a_{ij}]$ where $a_{ij} = 0$ when $i \neq j$).

4.3.4(L)

a. Let A and B denote any non-singular 2 by 2 matrices (i.e., A^{-1} and B^{-1} exist). Show that AB is also non-singular and in particular $(AB)^{-1} = B^{-1}A^{-1}$. (Note the order of the factors.)

b. Check (a) when $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$.

4.3.5

Let A denote the matrix

$$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 7 & 9 \\ 3 & 9 & 7 \end{pmatrix}$$

a. Use the augmented-matrix technique described in the notes to find A^{-1}.

b. If

$$y_1 = x_1 + 3x_2 + 5x_3$$

$$y_2 = 2x_1 + 7x_2 + 9x_3$$

$$y_3 = 3x_1 + 9x_2 + 7x_3$$

express x_1, x_2, x_3 as linear combinations of y_1, y_2, and y_3.

c. Solve the system of equations

$$x_1 + 3x_2 + 5x_3 = 8$$

$$2x_1 + 7x_2 + 9x_3 = 16$$

$$3x_1 + 9x_2 + 7x_3 = 32$$

4.3.6

Let $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 7 & 9 \\ 3 & 9 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 & 8 \\ 8 & 2 & 16 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 8 \\ 4 & 2 \\ 8 & 16 \end{bmatrix}$

a. Determine X if AX = C.

b. Determine Y if YA = B.

c. Repeat parts (a) and (b) with $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 8 \end{bmatrix}$.

d. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Show that there exist infinitely many matrices X such that $AX = I_2$.

e. With A, I_2, and I_3 as in part (d), show that there is no matrix X such that $XA = I_3$.

4.3.7

Find the flaw in the following argument. "Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Replace the first row of A by the sum of the first and the second rows; and replace the second row of A by the sum of the second row and the first row. Call the resulting matrix B. Therefore, $B = \begin{pmatrix} 4 & 6 \\ 7 & 10 \end{pmatrix}$. Since B was obtained from A by the 'permitted' operations described in the notes, it follows that $A \sim B$."