Topic 17

Modeling of Elasto-Plastic and Creep Response—Part I

Contents:

- **Basic considerations in modeling inelastic response**
- **A schematic review of laboratory test results, effects of stress level, temperature, strain rate**
- **One-dimensional stress-strain laws for elasto-plasticity, creep, and viscoplasticity**
- **Isotropic and kinematic hardening in plasticity**
- **General equations of multiaxial plasticity based on a yield condition, flow rule, and hardening rule**
- **Example of von Mises yield condition and isotropic hardening, evaluation of stress-strain law for general analysis**
- **Use of plastic work, effective stress, effective plastic strain**
- **Integration of stresses with subincrementation**
- **Example analysis: Plane strain punch problem**
- **Example analysis: Elasto-plastic response up to ultimate load of a plate with a hole**
- **Computer-plotted animation: Plate with a hole**

Textbook: Section 6.4.2

Example: 6.20

References: The plasticity computations are discussed in

Bathe, K. J., M. D. Snyder, A. P. Cimento, and W. D. Rolph III, "On Some Current Procedures and Difficulties in Finite Element Analysis of Elastic-Plastic Response," *Computers & Structures*, *12*, 607–624, 1980.

References:
(continued)

Snyder, M. D., and K. J. Bathe, "A Solution Procedure for Thermo-Elastic-Plastic and Creep Problems," *Nuclear Engineering and Design, 64,* 49–80, 1981.

The plane strain punch problem is also considered in

Sussman, T., and K. J. Bathe, "Finite Elements Based on Mixed Interpolation for Incompressible Elastic and Inelastic Analysis," *Computers & Structures,* to appear.

- WE DISCUSSED IN THE PREVIOUS LECTURES THE MODELING OF ELASTIC MATERIALS
  - LINEAR STRESS-STRAIN LAW
  - NONLINEAR STRESS-STRAIN LAW

  THE T.L. AND U.L. FORMULATIONS

- WE NOW WANT TO DISCUSS THE MODELING OF INELASTIC MATERIALS
  - ELASTO-PLASTICITY AND CREEP

- WE PROCEED AS FOLLOWS:
  - WE DISCUSS BRIEFLY INELASTIC MATERIAL BEHAVIORS, AS OBSERVED IN LABORATORY TESTS
  - WE DISCUSS BRIEFLY MODELING OF SUCH RESPONSE IN 1-D ANALYSIS
  - WE GENERALIZE OUR MODELING CONSIDERATIONS TO 2-D AND 3-D STRESS SITUATIONS

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MODELING OF INELASTIC RESPONSE: ELASTO-PLASTICITY, CREEP AND VISCOPLASTICITY

- The total stress is not uniquely related to the current total strain. Hence, to calculate the response history, stress increments must be evaluated for each time (load) step and added to the previous total stress.

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- The differential stress increment is obtained as – assuming infinitesimally small displacement conditions –

$$d\sigma_{ij} = C^E_{ijrs}\,(de_{rs} - de^{IN}_{rs})$$

where

C^E_{ijrs} = components of the elasticity tensor

de_{rs} = total differential strain increment

de^{IN}_{rs} = inelastic differential strain increment

The inelastic response may occur rapidly or slowly in time, depending on the problem of nature considered.

Modeling:

- In plasticity, the model assumes that de_{rs}^{IN} occurs instantaneously with the load application.
- In creep, the model assumes that de_{rs}^{IN} occurs as a function of time.
- The actual response in nature can be modeled using plasticity and creep together, or alternatively using a viscoplastic material model.

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— In the following discussion we assume small strain conditions, hence

- we have either a materially-nonlinear-only analysis
- or a large displacement/large rotation but small strain analysis

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- As pointed out earlier, for the large displacement solution we would use the total Lagrangian formulation and in the evaluation of the stress-strain laws simply use

  — Green-Lagrange strain component for the engineering strain components

  and

  — 2nd Piola-Kirchhoff stress components for the engineering stress components

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Consider a brief summary of some observations regarding material response measured in the laboratory

- We only consider schematically what approximate response is observed; no details are given.
- Note that, regarding the notation, no time, t, superscript is used on the stress and strain variables describing the material behavior.

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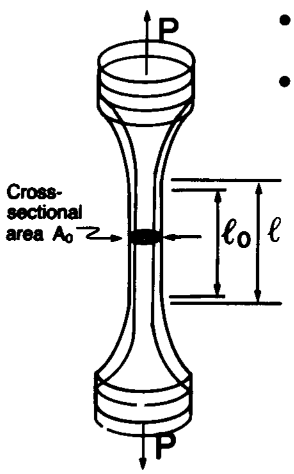
MATERIAL BEHAVIOR, "INSTANTANEOUS" RESPONSE

Tensile Test: Assume



- small strain conditions
- behavior in compression is the same as in tension

Hence

$$e = \frac{\ell - \ell_0}{\ell_0}$$

$$\sigma = \frac{P}{A_0}$$

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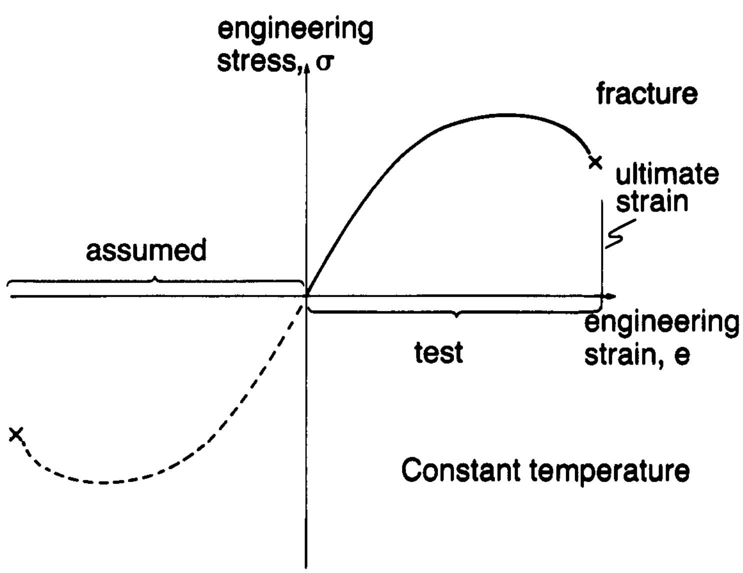

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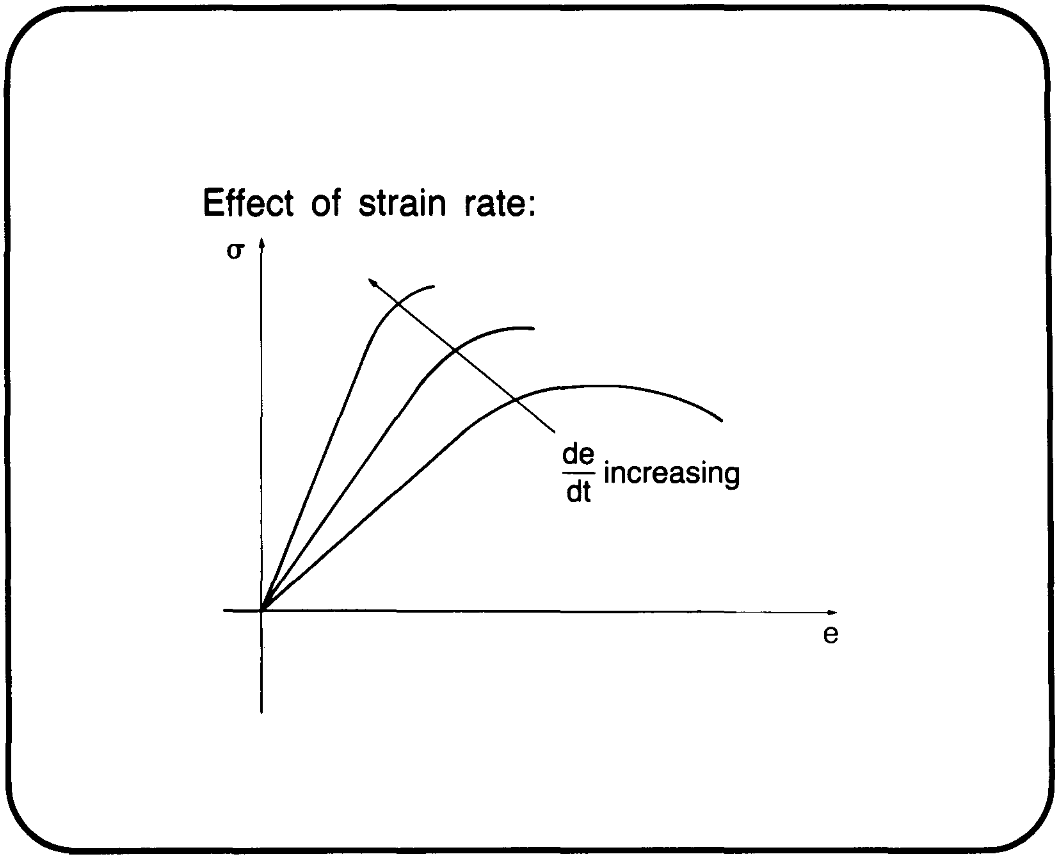

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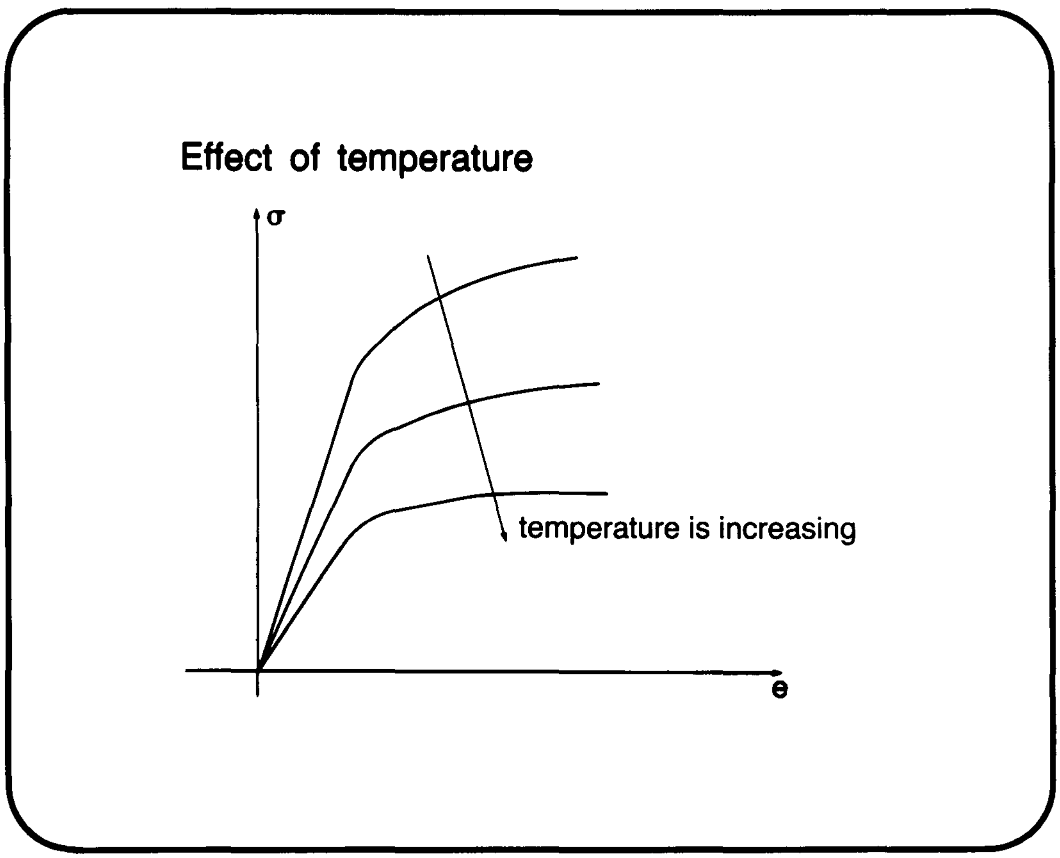

MATERIAL BEHAVIOR, TIME-DEPENDENT RESPONSE

- Now, at constant stress, inelastic strains develop.
- Important effect for materials when temperatures are high

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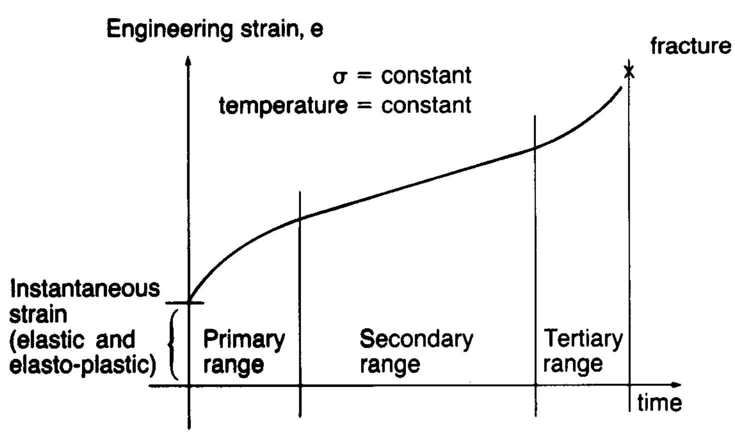
Typical creep curve



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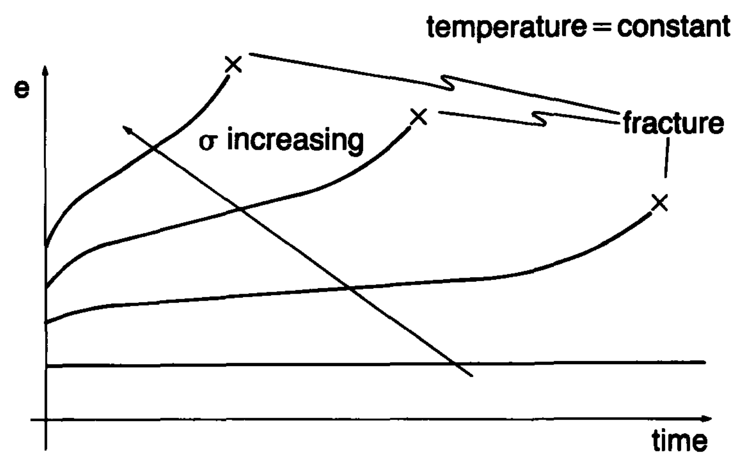
Effect of stress level on creep strain



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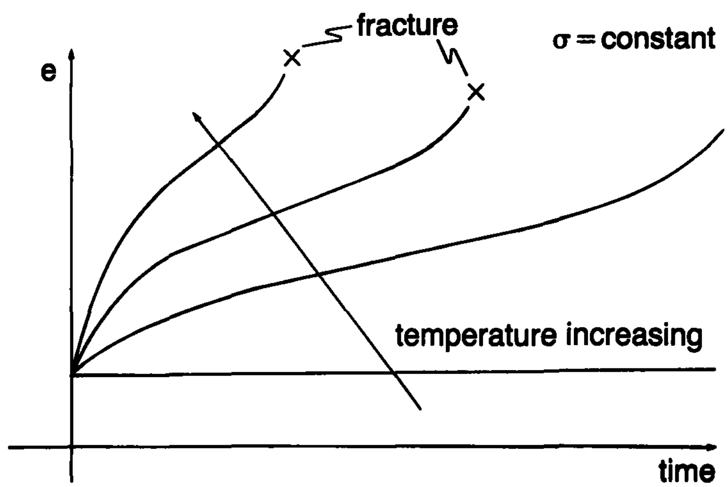
Effect of temperature on creep strain



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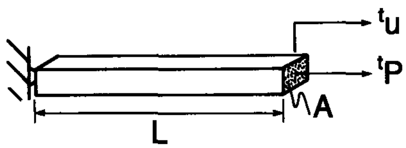
MODELING OF RESPONSE

Consider a one-dimensional situation:



$${}^t\sigma = \frac{{}^tP}{A}$$

$${}^te = \frac{{}^tu}{L}$$

- We assume that the load is increased monotonically to its final value, P*.
- We assume that the time is "long" so that inertia effects are negligible (static analysis).

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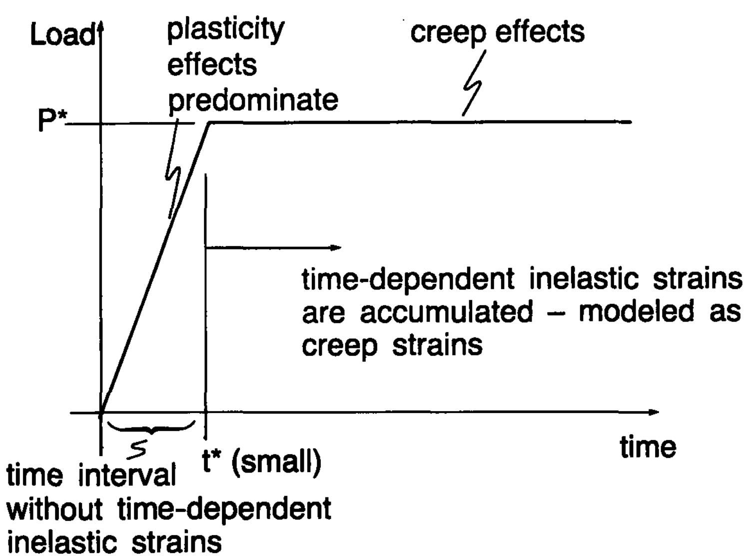

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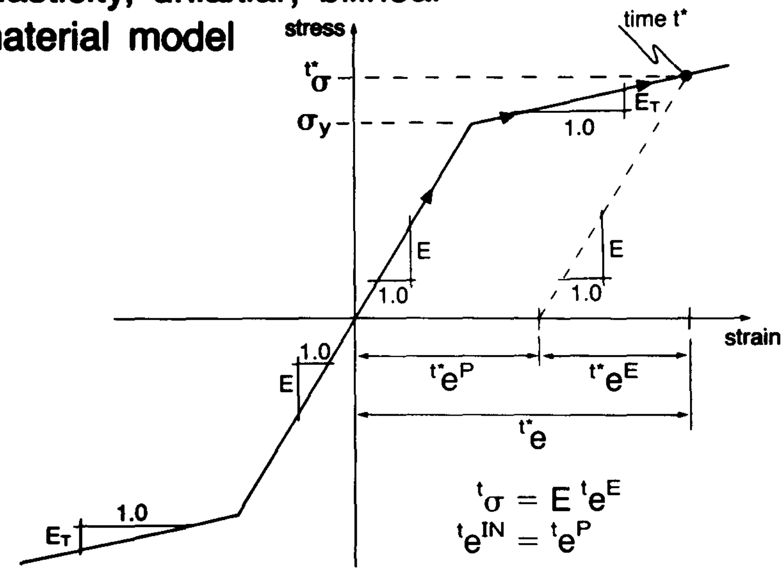
Plasticity, uniaxial, bilinear material model



$${}^{t}\sigma = E\,{}^{t}e^{E}$$

$${}^{t}e^{IN} = {}^{t}e^{P}$$

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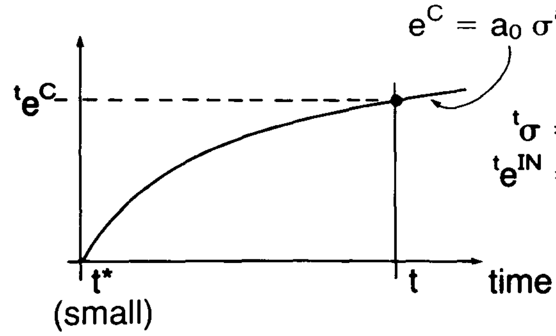
Creep, power law material model:

$$e^{C} = a_0\,\sigma^{a_1}\,t^{a_2}$$



$${}^{t}\sigma = E\,{}^{t}e^{E}$$

$${}^{t}e^{IN} = {}^{t}e^{C} + {}^{t}e^{P}$$

- The elastic strain is the same as in the plastic analysis (this follows from equilibrium).
- The inelastic strain is time-dependent and time is now an actual variable.

Viscoplasticity:

- Time-dependent response is modeled using a fluidity parameter γ:

$$\dot{e} = \frac{\dot{\sigma}}{E} + \underbrace{\gamma \left\langle \frac{\sigma}{\sigma_y} - 1 \right\rangle}_{\dot{e}^{VP}}$$

where

$$\langle \sigma - \sigma_y \rangle = \begin{cases} 0 & , \sigma \le \sigma_y \\ \sigma - \sigma_y & , \sigma > \sigma_y \end{cases}$$

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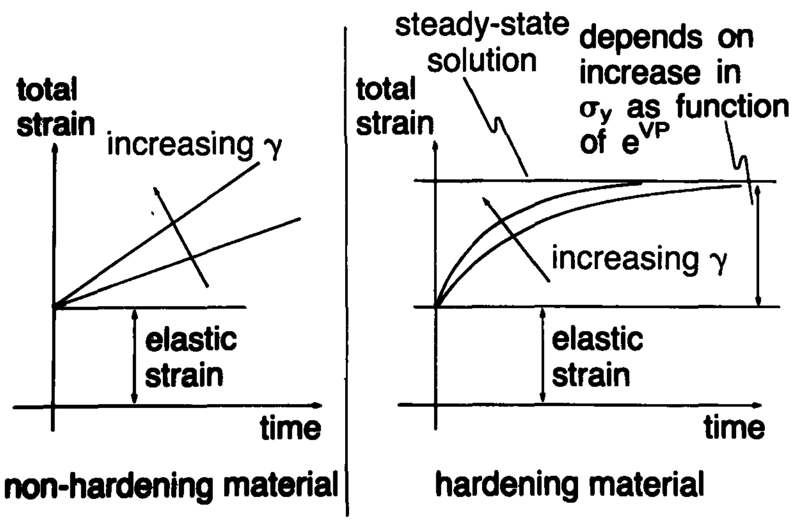
Typical solutions (1-D specimen):



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PLASTICITY

- So far we considered only loading conditions.
- Before we discuss more general multiaxial plasticity relations, consider unloading and cyclic loading assuming uniaxial stress conditions.

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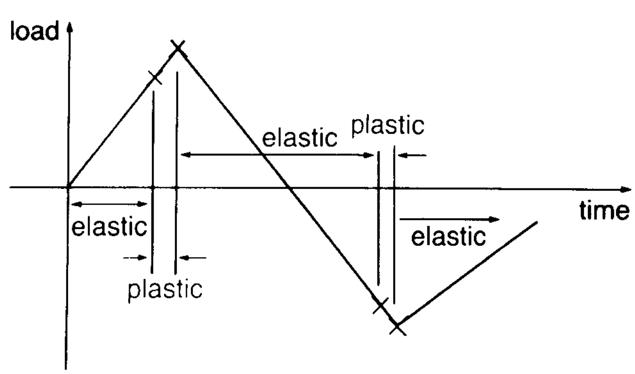
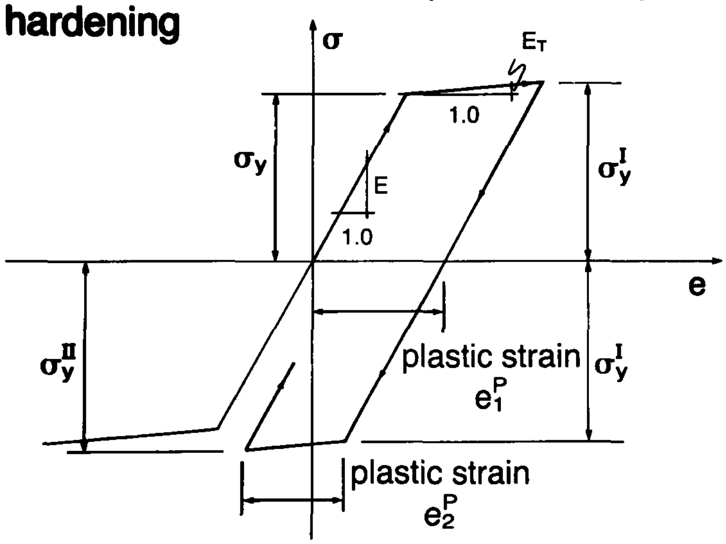
- Consider that the load increases in tension, causes plastic deformation, reverses elastically, and again causes plastic deformation in compression.



Bilinear material assumption, isotropic hardening



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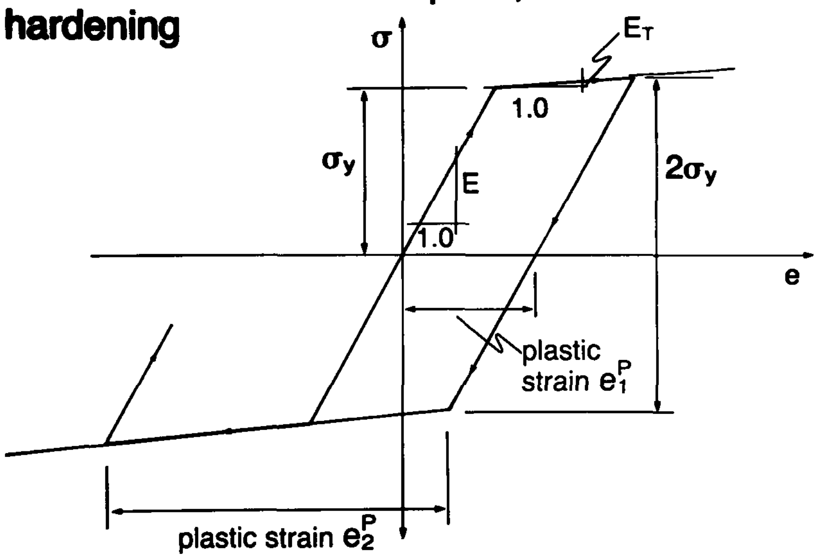
Bilinear material assumption, kinematic hardening



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MULTIAXIAL PLASTICITY

To describe the plastic behavior in multiaxial stress conditions, we use

- A yield condition
- A flow rule
- A hardening rule

In the following, we consider isothermal (constant temperature) conditions.

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These conditions are expressed using a stress function ${}^{t}F$.

Two widely used stress functions are the

von Mises function

Drucker-Prager function

von Mises

$$^tF = \frac{1}{2}\,{}^ts_{ij}\,{}^ts_{ij} - {}^t\kappa$$

$$^ts_{ij} = {}^t\sigma_{ij} - \frac{{}^t\sigma_{mm}}{3}\,\delta_{ij}\ ; \quad {}^t\kappa = \frac{1}{3}\,{}^t\sigma_y^2$$

Drucker-Prager

$$^tF = 3\alpha\,{}^t\sigma_m + {}^t\bar{\sigma} - k$$

$$^t\sigma_m = \frac{{}^t\sigma_{ii}}{3}\ ; \quad {}^t\bar{\sigma} = \sqrt{\frac{1}{2}\,{}^ts_{ij}\,{}^ts_{ij}}$$

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We use both matrix notation and index notation:

$$\underbrace{d\underline{e}^P = \begin{bmatrix} de_{11}^P \\ de_{22}^P \\ de_{33}^P \\ de_{12}^P + \boxed{de_{21}^P} \\ de_{23}^P + de_{32}^P \\ de_{13}^P + de_{31}^P \end{bmatrix}, \quad d\underline{\sigma} = \begin{bmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\sigma_{33} \\ d\sigma_{12} \\ d\sigma_{23} \\ d\sigma_{31} \end{bmatrix}}_{\text{matrix notation}}$$

note that both de_{12}^P and de_{21}^P are added

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$$de^P_{ij} = \begin{bmatrix} de^P_{11} & de^P_{12} & de^P_{13} \\ de^P_{21} & de^P_{22} & de^P_{23} \\ de^P_{31} & de^P_{32} & de^P_{33} \end{bmatrix}$$

$$d\sigma_{ij} = \begin{bmatrix} d\sigma_{11} & d\sigma_{12} & d\sigma_{13} \\ d\sigma_{21} & d\sigma_{22} & d\sigma_{23} \\ d\sigma_{31} & d\sigma_{32} & d\sigma_{33} \end{bmatrix}$$

index notation

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The basic equations are then (von Mises ${}^tF$):

1) Yield condition

$${}^tF\,({}^t\sigma_{ij},\ {}^t\kappa) = 0$$

current stresses — function of plastic strains

${}^tF$ is zero throughout the plastic response

- 1-D equivalent: $\frac{1}{3}\,({}^t\sigma^2 - {}^t\sigma_y^2) = 0$

  (uniaxial stress)

  current stresses — function of plastic strains.

2) Flow rule (associated rule):

$$de_{ij}^{P} = {}^{t}\lambda \, \frac{\partial {}^{t}F}{\partial {}^{t}\sigma_{ij}}$$

where ${}^{t}\lambda$ is a positive scalar.

- 1-D equivalent:

$$de_{11}^{P} = \frac{2}{3}\, {}^{t}\lambda \, {}^{t}\sigma$$

$$de_{22}^{P} = -\frac{1}{3}\, {}^{t}\lambda \, {}^{t}\sigma$$

$$de_{33}^{P} = -\frac{1}{3}\, {}^{t}\lambda \, {}^{t}\sigma$$

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3) Stress-strain relationship:

$$d\underline{\sigma} = \underline{C}^{E} (d\underline{e} - d\underline{e}^{P})$$

- 1-D equivalent:

$$d\sigma = E\,(de_{11} - de_{11}^{P})$$

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Our goal is to determine $\underline{C}^{EP}$ such that

$$d\underline{\sigma} = \underbrace{\underline{C}^{EP}}_{\text{instantaneous elastic-plastic stress-strain matrix}} d\underline{e}$$

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General derivation of $\underline{C}^{EP}$:

<u>Define</u>

$${}^{t}q_{ij} = \left.\frac{\partial\,{}^{t}F}{\partial\,{}^{t}\sigma_{ij}}\right|_{{}^{t}e_{ij}^{P}\ \text{fixed}}$$

$${}^{t}p_{ij} = -\left.\frac{\partial\,{}^{t}F}{\partial\,{}^{t}e_{ij}^{P}}\right|_{{}^{t}\sigma_{ij}\ \text{fixed}}$$

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Using matrix notation,

results from our definition of the plastic strain and stress increment vectors

$$ {}^t\underline{q}^T = [{}^tq_{11} \mid {}^tq_{22} \mid {}^tq_{33} \mid 2\,{}^tq_{12} \mid 2\,{}^tq_{23} \mid 2\,{}^tq_{31}] $$

$$ {}^t\underline{p}^T = [{}^tp_{11} \mid {}^tp_{22} \mid {}^tp_{33} \mid {}^tp_{12} \mid {}^tp_{23} \mid {}^tp_{31}] $$

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We now determine ${}^t\lambda$ in terms of $d\underline{e}$:

Using ${}^tF = 0$ during plastic deformations,

$$ d\,{}^tF = \frac{\partial\, {}^tF}{\partial\, {}^t\sigma_{ij}}\, d\sigma_{ij} + \frac{\partial\, {}^tF}{\partial\, {}^te^P_{ij}}\, de^P_{ij} $$

$$ = {}^t\underline{q}^T\, d\underline{\sigma} - {}^t\underline{p}^T \underbrace{d\underline{e}^P}_{{}^t\lambda\, {}^t\underline{q}} $$

$$ = 0 $$

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Also

$$ {}^t\underline{q}^T \, \underbrace{d\underline{\sigma}} = {}^t\underline{q}^T \, \underbrace{(\underline{C}^E \, (d\underline{e} - d\underline{e}^P))} $$

The flow rule assumption may be written as

$$ d\underline{e}^P = {}^t\lambda \, {}^t\underline{q} $$

Hence

$$ {}^t\underline{q}^T \, d\underline{\sigma} = \boxed{{}^t\underline{q}^T \, (\underline{C}^E \, (d\underline{e} - {}^t\lambda \, {}^t\underline{q})) = \underbrace{{}^t\lambda \, {}^t\underline{p}^T \, {}^t\underline{q}}_{\text{from } d\,{}^tF = 0}} $$

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Solving the boxed equation for ${}^t\lambda$ gives

$$ {}^t\lambda = \frac{{}^t\underline{q}^T \, \underline{C}^E \, d\underline{e}}{{}^t\underline{p}^T \, {}^t\underline{q} + {}^t\underline{q}^T \, \underline{C}^E \, {}^t\underline{q}} $$

Hence we can determine the plastic strain increment from the total strain increment:

$$ \underbrace{d\underline{e}^P}_{\text{plastic strain increment}} = \left(\frac{{}^t\underline{q}^T \, \underline{C}^E \, \overbrace{d\underline{e}}^{\text{total strain increment}}}{{}^t\underline{p}^T \, {}^t\underline{q} + {}^t\underline{q}^T \, \underline{C}^E \, {}^t\underline{q}} \right) {}^t\underline{q} $$

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We can now solve for $\underline{C}^{EP}$:

$$d\underline{\sigma} = \underline{C}^{E}\,(d\underline{e} - d\underline{e}^{P})$$

function of $d\underline{e}$ from above

$$\downarrow$$

$$\underline{C}^{EP} = \underline{C}^{E} - \frac{\underline{C}^{E}\,{}^{t}\underline{q}\,(\underline{C}^{E}\,{}^{t}\underline{q})^{T}}{{}^{t}\underline{p}^{T}\,{}^{t}\underline{q} + {}^{t}\underline{q}^{T}\,\underline{C}^{E}\,{}^{t}\underline{q}}$$

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Example: Von Mises yield condition, isotropic hardening

Two equivalent equations:

$$ {}^{t}\sigma_y = \frac{\sqrt{2}}{2}\sqrt{({}^{t}\sigma_1 - {}^{t}\sigma_2)^2 + ({}^{t}\sigma_2 - {}^{t}\sigma_3)^2 + ({}^{t}\sigma_3 - {}^{t}\sigma_1)^2}$$

principal stresses

$$ {}^{t}F = \frac{1}{2}\,{}^{t}s_{ij}\,{}^{t}s_{ij} - {}^{t}\kappa \;;\quad {}^{t}\kappa = \frac{1}{3}\,{}^{t}\sigma_y^2$$

deviatoric stresses: ${}^{t}s_{ij} = {}^{t}\sigma_{ij} - \frac{{}^{t}\sigma_{mm}}{3}\,\delta_{ij}$

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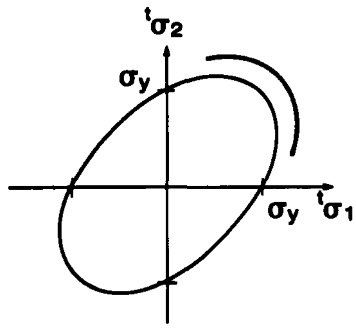
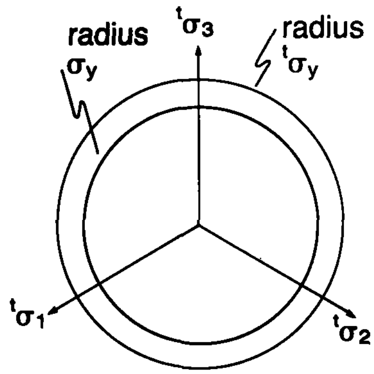
Yield surface for plane stress



End view of yield surface



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We now compute the derivatives of the yield function.

First consider ${}^tp_{ij}$:

$$ {}^tp_{ij} = - \left.\frac{\partial\, {}^tF}{\partial\, {}^te^P_{ij}}\right|_{{}^t\sigma_{ij}\ \text{fixed}} = - \frac{\partial}{\partial\, {}^te^P_{ij}} \left(\frac{1}{2}\, {}^ts_{ij}\, {}^ts_{ij} - \frac{1}{3}\, {}^t\sigma_y^2 \right) $$

$$ = \frac{2}{3}\, {}^t\sigma_y \frac{\partial\, {}^t\sigma_y}{\partial\, {}^te^P_{ij}} $$

(${}^t\sigma_{ij}$ fixed implies ${}^ts_{ij}$ is fixed)

What is the relationship between ${}^{t}\sigma_y$ and the plastic strains?

We answer this question using the concept of "plastic work".

- The plastic work (per unit volume) is the amount of energy that is unrecoverable when the material is unloaded.
- This energy has been used in creating the plastic deformations within the material.

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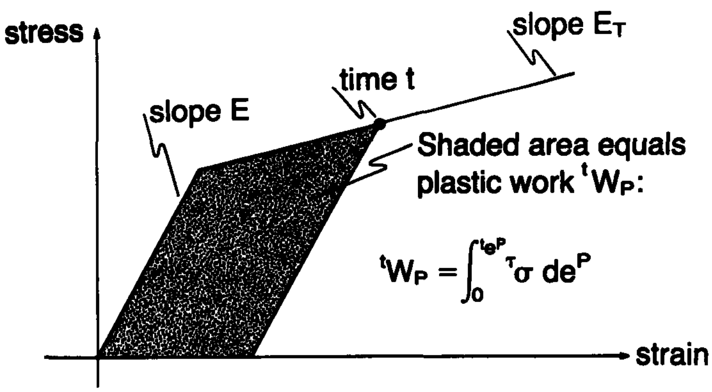
- **Pictorially: 1-D example**



- **In general,** $\mathbf{{}^{t}W_P = \int_0^{{}^{t}e^P_{ij}} {}^{\tau}\sigma_{ij} \, de^P_{ij}}$

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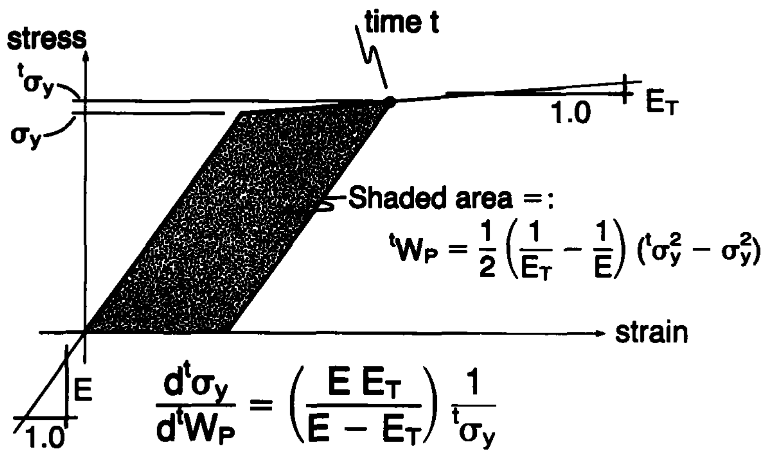
Consider 1-D test results: the current yield stress may be written in terms of the plastic work.



$$\frac{d\,{}^t\sigma_y}{d\,{}^tW_P} = \left(\frac{E\,E_T}{E - E_T}\right)\frac{1}{{}^t\sigma_y}$$

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We can now evaluate ${}^tp_{ij}$ – which corresponds to a generalization of the 1-D test results to multiaxial conditions.

$$ {}^tp_{ij} = \frac{2}{3}\,{}^t\sigma_y \underbrace{\left(\frac{d\,{}^t\sigma_y}{d\,{}^tW_P}\,\frac{\partial\,{}^tW_P}{\partial\,{}^te_{ij}^P}\right)}_{\frac{\partial\,{}^t\sigma_y}{\partial\,{}^te_{ij}^P}} $$

$$ = \frac{2}{3}\,{}^t\sigma_y \left(\left(\frac{E\,E_T}{E - E_T}\right)\frac{1}{{}^t\sigma_y}\right)({}^t\sigma_{ij}) $$

$$ = \boxed{\frac{2}{3}\left(\frac{E\,E_T}{E - E_T}\right){}^t\sigma_{ij}} $$

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Alternatively, we could have used that

$$d\,{}^{t}W_P = {}^{t}\bar{\sigma}\, d\,{}^{t}\bar{e}^P$$

where

$${}^{t}\bar{\sigma} = \sqrt{\frac{3}{2}\,{}^{t}s_{ij}\,{}^{t}s_{ij}} \quad \text{(effective stress)}$$

$$d\,{}^{t}\bar{e}^P = \sqrt{\frac{2}{3}\,de_{ij}^P\,de_{ij}^P}\,\Bigg|_t \quad \text{(increment in effective plastic strain)}$$

and then the same result is obtained using

$${}^{t}p_{ij} = \frac{2}{3}\,{}^{t}\sigma_y \left(\frac{d\,{}^{t}\sigma_y}{d\,{}^{t}\bar{e}^P}\,\frac{\partial\,{}^{t}\bar{e}^P}{\partial\,{}^{t}e_{ij}^P}\right)$$

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Next consider ${}^{t}q_{ij}$:

$${}^{t}q_{ij} = \frac{\partial\,{}^{t}F}{\partial\,{}^{t}\sigma_{ij}}\bigg|_{{}^{t}e_{ij}^P\ \text{fixed}} = \frac{\partial}{\partial\,{}^{t}\sigma_{ij}}\left(\frac{1}{2}\,{}^{t}s_{k\ell}\,{}^{t}s_{k\ell} - \frac{1}{3}\,{}^{t}\sigma_y^2\right)$$

$$= {}^{t}s_{k\ell}\,\frac{\partial\,{}^{t}s_{k\ell}}{\partial\,{}^{t}\sigma_{ij}} = {}^{t}s_{k\ell}\,\frac{\partial}{\partial\,{}^{t}\sigma_{ij}}\left({}^{t}\sigma_{k\ell} - \frac{{}^{t}\sigma_{mm}}{3}\,\delta_{k\ell}\right)$$

$$= {}^{t}s_{k\ell}\left(\delta_{ik}\,\delta_{j\ell} - \frac{\delta_{ij}\,\delta_{k\ell}}{3}\right)$$

$$= {}^{t}s_{ij} \quad (\text{note that } {}^{t}s_{k\ell}\,\delta_{k\ell} = {}^{t}s_{kk} = 0)$$

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We can now evaluate $\underline{C}^{EP}$:

$$
\underline{C}^{EP} = \frac{E}{1+\nu}
\begin{array}{c}
\begin{array}{ccccc} de_{11} & de_{22} & & 2\,de_{12} & \end{array} \\
\begin{bmatrix}
\dfrac{1-\nu}{1-2\nu} - \beta({}^t s_{11})^2 & \dfrac{\nu}{1-2\nu} - \beta\, {}^t s_{11}\, {}^t s_{22} & \cdots & -\beta\, {}^t s_{11}\, {}^t s_{12} & \cdots \\
 & \dfrac{1-\nu}{1-2\nu} - \beta({}^t s_{22})^2 & \cdots & -\beta\, {}^t s_{22}\, {}^t s_{12} & \cdots \\
 & & \cdots & -\beta\, {}^t s_{33}\, {}^t s_{12} & \cdots \\
 & \text{symmetric} & & \dfrac{1}{2} - \beta({}^t s_{12})^2 & \cdots \\
 & & & & \cdots
\end{bmatrix}
\end{array}
$$

$$
\text{where } \beta = \frac{3}{2}\frac{1}{{}^t\sigma_y^2}\left(\frac{1}{1 + \dfrac{2}{3}\dfrac{E\,E_T}{E - E_T}\dfrac{1+\nu}{E}}\right)
$$

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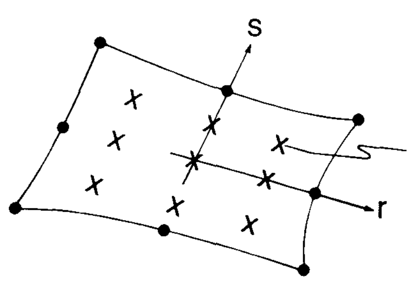
Evaluation of the stresses at time $t+\Delta t$:

$$
{}^{t+\Delta t}\underline{\sigma} = {}^t\underline{\sigma} + \int_t^{t+\Delta t} d\underline{\sigma}
$$

$$
= {}^t\underline{\sigma} + \int_{{}^t\underline{e}}^{{}^{t+\Delta t}\underline{e}} \underline{C}^{EP}\, d\underline{e}
$$



The stress integration must be performed at each Gauss integration point.

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We can approximate the evaluation of this integral using the Euler forward method.

- Without subincrementation:

$$\int_{{}^{t}\underline{e}}^{{}^{t+\Delta t}\underline{e}} \underline{C}^{EP}\, d\underline{e} \doteq \underline{C}^{EP}\Big|_{t} \, \underbrace{\Delta \underline{e}}_{} \qquad {}^{t+\Delta t}\underline{e} - {}^{t}\underline{e}$$

Transparency 17-52

- With n subincrements:

$$\int_{{}^{t}\underline{e}}^{{}^{t+\Delta t}\underline{e}} \underline{C}^{EP}\, d\underline{e} \doteq \underline{C}^{EP}\Big|_{t} \frac{\Delta \underline{e}}{n}$$

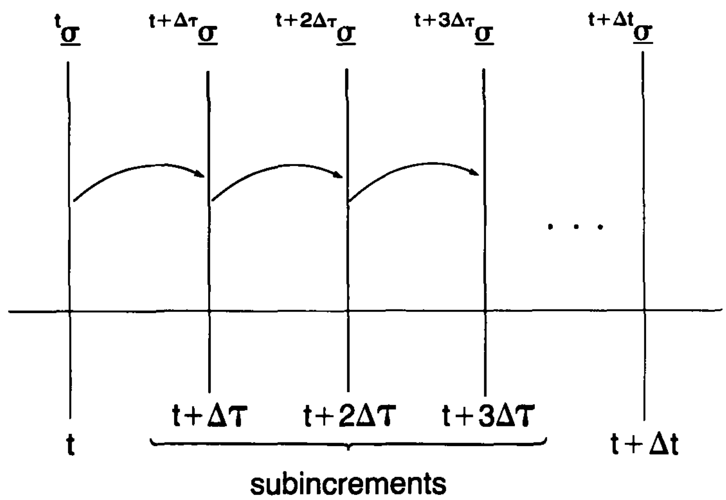
$$+ \underline{C}^{EP}\Big|_{t+\underline{\Delta\tau}} \frac{\Delta \underline{e}}{n} \qquad \frac{\Delta t}{n}$$

$$+ \cdots$$

$$+ \underline{C}^{EP}\Big|_{t+(n-1)\Delta\tau} \frac{\Delta \underline{e}}{n}$$

Transparency 17-53

Pictorially:



Transparency 17-54

Summary of the procedure used to calculate the total stresses at time $t+\Delta t$.

Given:

STRAIN = Total strains at time $t+\Delta t$
SIG = Total stresses at time t
EPS = Total strains at time t

(a) Calculate the strain increment DELEPS:

DELEPS = STRAIN − EPS

(b) Calculate the stress increment DELSIG, assuming elastic behavior:

$$\text{DELSIG} = C^E * \text{DELEPS}$$

(c) Calculate TAU, assuming elastic behavior:

$$\text{TAU} = \text{SIG} + \text{DELSIG}$$

(d) With TAU as the state of stress, calculate the value of the yield function F.

(e) If F(TAU) ≤ 0, the strain increment is elastic. In this case, TAU is correct; we return.

Transparency 17-55

(f) If the previous state of stress was plastic, set RATIO to zero and go to (g). Otherwise, there is a transition from elastic to plastic and RATIO (the portion of incremental strain taken elastically) has to be determined. RATIO is determined from

$$F\,(\text{SIG} + \text{RATIO} * \text{DELSIG}) = 0$$

since $F = 0$ signals the initiation of yielding.

Transparency 17-56

Transparency 17-57

(g) Redefine TAU as the stress at start of yield

$$TAU = SIG + RATIO * DELSIG$$

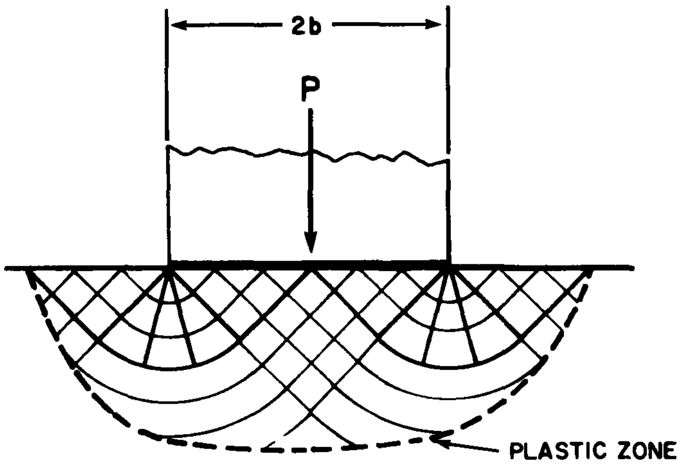
and calculate the elastic-plastic strain increment

$$DEPS = (1 - RATIO) * DELEPS$$

(h) Divide DEPS into subincrements DDEPS and calculate

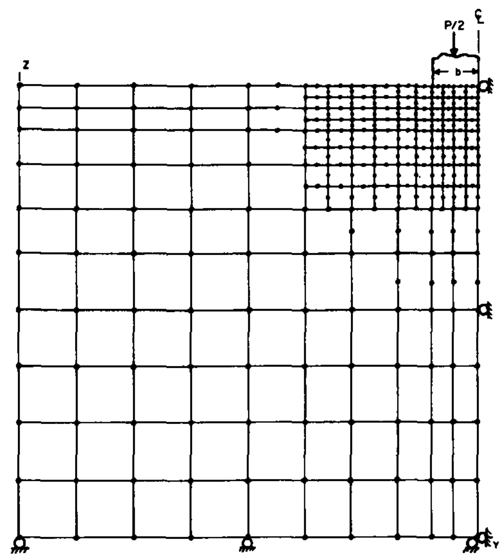
$$TAU \leftarrow TAU + \underline{C}^{EP} * DDEPS$$

for all elastic-plastic strain subincrements.



Plane strain punch problem

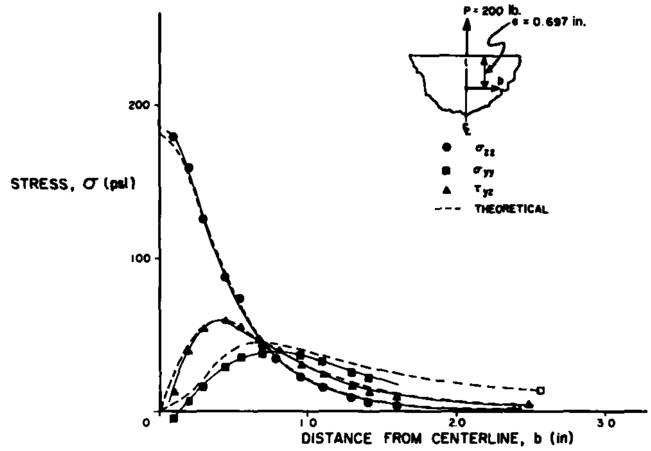
Slide 17-1



Finite element model of punch problem

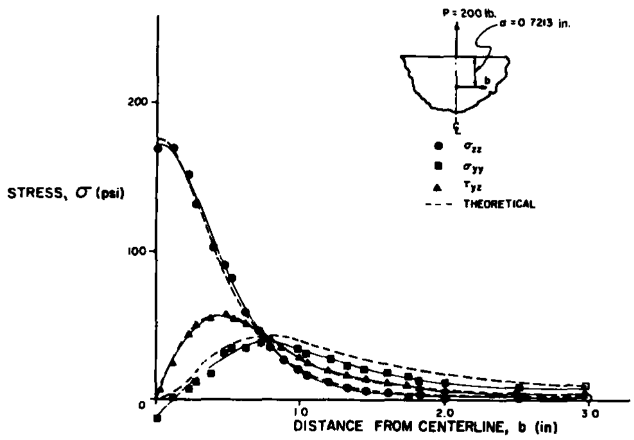
Slide 17-2

Slide 17-3

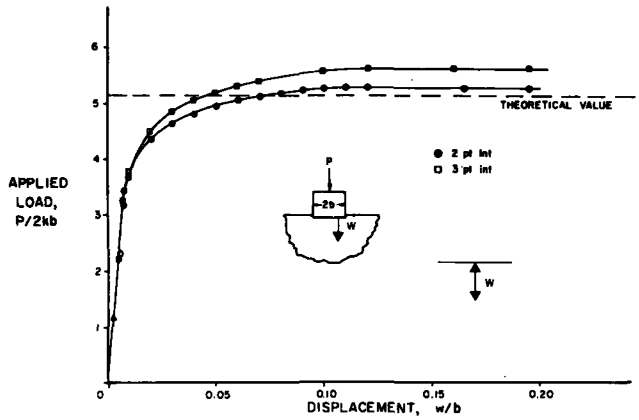


Solution of Boussinesq problem—2 pt. integration

Slide 17-4



Solution of Boussinesq problem—3 pt. Integration

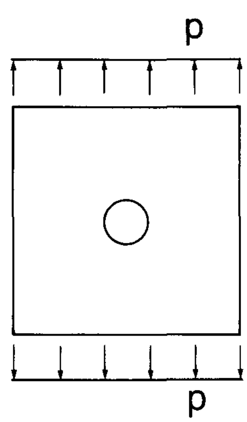


Load-displacement curves for punch problem

Slide 17-5

Transparency 17-58

Limit load calculations:

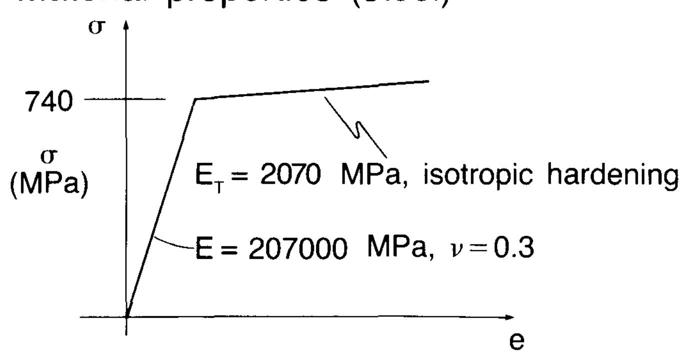


• Plate is elasto-plastic.

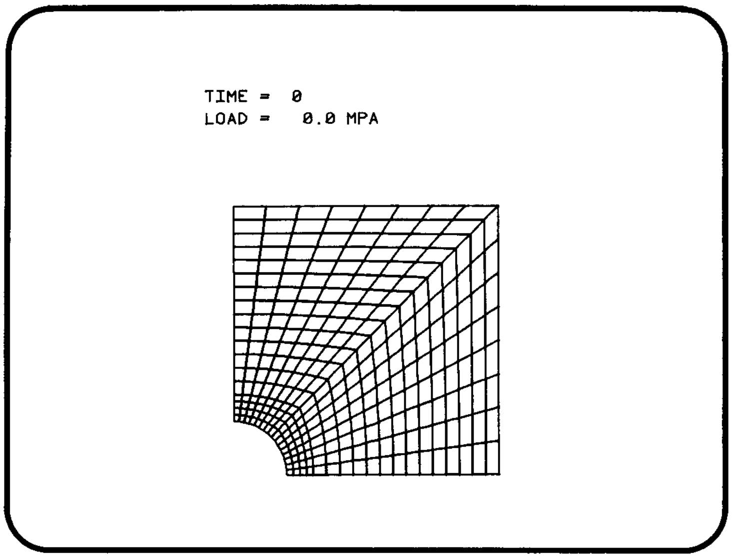
Transparency 17-59

Elasto-plastic analysis:

Material properties (steel)



• This is an idealization, probably inaccurate for large strain conditions ($e > 2\%$).



Computer Animation
Plate with hole