PROFESSOR: In this lecture we begin a discussion of the topic of modulation, which is, among other things, a very important topic in practical terms. For example, it forms the cornerstone for many communication systems. And also, as we'll see as these lectures go along, a particular form of modulation referred to as pulse amplitude modulation, and eventually impulse modulation or impulse train modulation, forms a very important bridge between continuous time signals and discrete time signals.

Now in general terms what we mean when we refer to modulation is the notion of using one signal to vary a parameter of another signal. For example, a sinusoidal signal has three parameters, amplitude, frequency, and phase. And we could think, for example, of using one signal to vary, let's say, the amplitude of a sinusoidal signal. And what that leads to is a notion, which we'll develop in some detail, referred to as sinusoidal amplitude modulation, and would correspond to a sinusoidal signal, referred to as the carrier, and it's amplitude being varied on the basis of another signal.

Now alternatively we could think of varying either the frequency or the phase of a sinusoidal signal, again with another signal. And what that leads to is another very important notion, which is referred to sinusoidal frequency modulation, where essentially it's the frequency of a sinusoid that's changing depending on the signal that's we're using to modulate the sinusoid.

Now sinusoidal amplitude, frequency, and phase modulation are extremely important topics and ideas in the context of communication systems. One of the reasons is that if you want to transmit a signal, let's say for example a voice signal, the voice signal that you're listening to now. If you try to transmit that over long distances, because of the frequencies involved the medium that you use to transmit it won't carry it long distances. The idea then is to essentially take that signal, like a voice signal, use it to modulate a much higher-frequency signal, and then transmit that higher-frequency signal over a medium that essentially can support long-distance transmission at those frequencies. Then at the other end of course, the voice information, or whatever else the information is, is taken off.

Now also, a notion that that leads to, and we'll be developing in some detail, is the idea that you can simultaneously transmit more than one signal by in essence taking several voice signals or other signals, using them to modulate either the frequency or amplitude of sinusoidal signals at different frequencies, adding all those together-- that's a process called multiplexing-- and then at the other end of the transmission system, taking those sinusoidal signals apart. And then extracting the envelope or frequency modulation information to get back to the voice signal or other information-carrying signal.

So that's one of the very important ways in which sinusoidal modulation is used in communication systems. And what we'll see, in particular as we go through today's lecture, is that sinusoidal amplitude modulation, follows in a fairly straightforward way from the properties of the Fourier transform that we've developed in some of the earlier lectures.

So our focus in today's lecture will be on sinusoidal amplitude modulation in continuous time. In the next lecture we'll consider the same set of notions related to discrete time, and also a concept referred to as pulse amplitude modulation. And all of these follow, in a very straightforward way, from the modulation property for the Fourier transform.

Issues of frequency and phase modulation are a little more difficult to analyze. But many of the techniques that we've developed in the previous lectures also provide important insights into frequency and phase modulation. And some of this is developed in more detail in the book.

So what I'd like to do is focus, for now, on the concept of amplitude modulation. And as I indicated, there are several kinds of carrier signals on which the modulation can be superimposed. The basic structure for an amplitude modulation system is one in which there is the modulating signal, let's say for example, voice, and a carrier signal-- what's referred to as the carrier. And then of course the resulting output is the modulated output.

Now to analyze this, since we have multiplication in the time domain, we know from the property of the Fourier transform that we've developed previously-- the modulation property-- that multiplication in the time domain corresponds to convolution in the frequency domain. And it's this basic property or equation that lets us analyze, in some detail in fact, the notions of amplitude modulation.

As we go through this lecture and the next lecture, we'll be talking, as I indicated, about several different types of carrier signals. One is what's referred to as pulse carriers. And that leads to, among other things, the concept of pulse amplitude modulation. That will be deferred until the next lecture.

In today's lecture what I'll focus on is first, the case of a complex exponential carrier, second, the case of sinusoidal carrier. And in fact the complex exponential carrier and sinusoidal carrier are obviously very closely related, since the complex exponential carrier is, in effect, two sinusoidal carriers. One for the real part and one for the imaginary part. So let's first begin the discussion of amplitude modulation by considering a complex exponential carrier, and then moving on to a discussion of a sinusoidal carrier.

So the issue then is that we have a signal, x of t. It's multiplied by a carrier. And the carrier that we're considering is a carrier signal, c of t, of the form e to the j omega c t plus theta sub c. That's the form of our carrier signal.

And what we can first analyze is what the resulting signal or spectrum is at the output of the modulator. Well, we can do that by concentrating on the modulation property. And let's consider, just as a general form for a spectrum, what I've indicated here for the Fourier transform of the input signal or modulating signal, X of omega. And so this is intended to represent the spectrum of x of t.

And then the carrier signal, since it's a single complex exponential, has a Fourier transform which is an impulse in the frequency domain. And the amplitude of the impulse is 2 pi e to the j theta sub c, where we notice that the complex amplitude incorporates the phase information.

So now if we multiply in the time domain, we convolve in the frequency domain. And as you know, convolving a signal with an impulse just shifts that signal to the location of the impulse. And so as a consequence of taking care of various factors, what we end up with is a spectrum that is centered at the carrier frequency omega sub c.

So what this says is that if we have a signal, x of t, and we use it to modulate a complex exponential carrier in the frequency domain, what we've simply done is to take the original spectrum and shift it in frequency. So that what was originally at zero frequency is now centered around the carrier frequency.

We've now modulated, in effect, to a higher frequency. Things are happening in a higher-frequency band. And the next question is, how do we demodulate, or in other words, how do we get the original signal back?

Of course one way that we can think of doing it, particularly in the context of this specific carrier, if we look back at the top equation we have, as the result of the modulation, x of t times c of t, where c of t is this. And we could consider, for example, just simply dividing the modulator output by this. Or equivalently, taking the modulated output and multiplying by e to the minus j omega c t plus theta c.

Let's track that through in terms of the spectra. We have, again, the spectrum of the output of the modulator, which is the original spectrum shifted up to the carrier frequency. We have, below that, the spectrum of e to the minus j omega c t plus theta c. And if we now convolve this with this, that results in simply shifting this spectrum-- except for an issue of a scale factor-- shifting this spectrum back down to the origin. So convolving these two together, the spectrum that we end up with is that.

So we can track this through in the frequency domain. In the frequency domain it says shift the spectrum up. When you want to demodulate, shift the spectrum back down. And alternatively, we can look at it algebraically in the time domain. And what it says is, if you multiply by e to the plus j omega c t, then when you want to get back, multiply by e to the minus j omega c t.

Now one question that you could conceivably be asking is, if we're talking about practical systems and not simply mathematics, does it make sense in the real world to consider using a complex exponential carrier? And the answer to that, in fact, is yes. That very often in practical systems one considers using a carrier which in fact is a complex exponential.

Well, a complex exponential is complex. There's a square root of minus one in there. And you could ask well, how do we get a square root of minus one? And the answer is fairly simple.

Let's look again at the modulator, which we have here. And in effect, what that says is we want to multiply a real-valued signal by e to the j omega c t plus theta c. Now, we can equivalently use Euler's relationship to break this down into a cosine and sine term.

And so what that means in terms of an implementation, equivalently, is modulating x of t onto a cosine carrier. And that then gives us the real part of the complex output. And modulating it onto a sinusoidal carrier-- these two being 90 degrees out of phase-- and that gives us the imaginary part. And so in effect, this is the complex signal.

If we just simply think of hanging a tag on here that says square root of minus 1, or j, and we appropriately combine complex signals following the rules of complex arithmetic. And indeed, that's exactly the way things are done in the real world. A complex signal is simply a set of two real signals.

And of course, if we look at the spectra involved, we have here the real part and the imaginary part of the complex output. If we again refer back to the original spectrum, X of omega, and the modulated spectrum which I show down here, the original spectrum shifted up to the carrier frequency. In effect we're building this out of two lines. One line representing the real part of that. And the real part in the time domain corresponds to the even part in the frequency domain. And so with the output of the cosine modulator, we have a spectrum that looks like this. And the output along the imaginary branch has a spectrum that looks like this.

Recall in the top branch that this, for positive frequencies was positive, and was positive here. And so in effect when you add them, this portion of the spectrum will cancel out. So in effect, what we're doing is building the complex signal out of two real signals. Or we're building the spectrum of the complex signal out of separate lines that represent the even and the odd parts.

Now, there are lots of applications of amplitude modulation. And we'll be seeing a number of these as we go through the discussion. What I'd like to do is just indicate briefly one now, which is an application that in fact surfaces fairly often in the context of a complex exponential carrier. And that is the notion of using modulation to permit the application of a very well designed and implemented low-pass filter to be used as a band-pass filter and in fact, as a set of band-pass filters.

And here's the idea. The idea is if we have a fixed filter-- let's say we have a signal. And we want to think of a filter, which we want to move along the signal, one way to do it is to somehow have filters that move along the signal. The other possibility is to keep the filter fixed and let the signal move in frequency in front of the filter.

Let me be a little more specific. Suppose that we have a signal, x of t. And we modulate it with a complex exponential carrier with a carrier frequency, omega c. And the output of that is then processed with a low-pass filter and then we demodulate the result. Then what we've done is to take the spectrum of the input signal, shift it, pull out what is now around low frequencies, and then shift that part of the spectrum back to where it belongs.

So if we look at that in terms of actually tracking through the spectra, we would have initially a spectrum for the original signal, which I show at the top as X of omega.

After modulating or shifting that spectrum up to a center frequency of omega c, we then have what I indicate here. And the dotted line corresponds to the pass band of the low-pass filter. Well, the result of low-pass filtering rejects all the spectrum except the part around low frequencies.

And the next step is then to demodulate this. And so in effect, demodulating will shift this spectrum back to where it originally came from. And so that result will be what I show in the final result, which is here. And what we can see is that this is equivalent. If we can look back at the top spectrum, this is equivalent to having extracted, with a band-pass filter, a section out of this part of the spectrum.

So in terms of tracking through the spectrum and looking at the equivalent filtering operation, then what we accomplished was to pull out this part of the spectrum using a low-pass filter and modulation. But equivalently what we implemented was a band-pass filter as I indicated here. Now of course, a signal with this spectrum, since the spectrum is not conjugate symmetric, we know that this signal does not correspond to a real-valued signal. Equivalently this filter doesn't correspond to a filter whose impulse response is real.

If we add another step to this, which is to take the real part of the output, then by taking the real part of the output we would be taking the even part of the spectrum associated with that complex signal. And the equivalent filter that we would end up with then is the filter that I indicate at the bottom, which is a band-pass filter.

Now just to reiterate a point that I made earlier. A question, of course, is why would you go to this trouble? Why not just build a band-pass filter? And one of the reasons is that it's often much easier to build a fixed filter, a filter with a fixed-center frequency, for example a low-pass filter, than it is to build a filter that has variable components in it so that when you vary them the filter's center frequency shifts around.

Now, if you want to look at the energy in a signal in different frequency bands, then you'd like to look at it through different filters. And so the idea here, which is really the basis for many spectrum analyzers, is to build a really good quality low-pass filter and then use modulation, which is often easier to implement. Use modulation to shift the signal essentially in front of the filter.

So we've worked our way through modulation with a complex exponential carrier. And what we saw, among other things with a complex exponential carrier, is that what it corresponds to is two branches. One being modulation with a cosine, and the other, modulation with a sine. And so in the real world, or in a practical system, modulation of the complex exponential carrier really would be accomplished with modulation with a sinusoidal carrier, and in particular with sinusoidal carriers that are in quadrature, as it's referred to, or equivalently 90 degrees out of phase.

Well, in fact sinusoidal modulation, in other words, modulation using only a sinusoidal carrier, very often is used in its own right not only for generating a complex exponential carrier, but as a carrier by itself. Let's look at what the consequences of modulation with a sinusoidal carrier are. And in particular work through, again, what the spectra are and how we get the original signal back again.

So we are talking about a carrier signal which is simply a sinusoidal signal with some phase. And of course we can write that as the sum of two complex exponential signals. And so now, when we apply the modulation property we have the original spectrum, which I show here, X of omega. And that's convolved with the spectrum of the carrier. And the spectrum of the carrier, in this case, is two impulses. One at plus omega c, and one at minus omega c. And the amplitudes of these incorporate the phase.

And later on in the lecture, and in subsequent lectures, I'll have a tendency to drop the theta sub c, just to keep the notation and algebra a little cleaner, but for now I've incorporated it. And so now when we apply the modulation property, what we will do is convolve this spectrum with this spectrum, and the result is that the spectrum of the original signal gets replicated at both omega sub c and at minus omega sub c. And the resulting spectrum at the output of the modulator, then, is the spectrum that I show here.

Now the question, of course, is-- so now what's happened is that with a sinusoidal carrier, we've moved the spectrum to both plus omega c and minus omega c. And now if we want to get the original signal back again, what we would like to do somehow is move that spectrum back down to the origin. Now in the case of a complex exponential, that was easy to do. We'd shifted one up, we'd just shift it back down. Let's see what happens if we attempt to demodulate by again multiplying by the same sinusoidal carrier.

So let's examine what happens if we now take our modulated signal and, again, modulate it onto the same sinusoidal carrier to generate the output w of t. If we look at the spectra, we have the modulated spectrum which we had initially. And we now want to convolve that, again, with the spectrum of the carrier signal. The spectrum of the carrier signal, I indicate here.

And if you track through the convolution, which is fairly straightforward, then what happens as you convolve this with this is you end up with a composite spectrum, which is what I've indicated on the bottom curve, and has the spectrum of the original signal, x of t, replicated in three places. One is at minus 2 omega sub c. One is around the origin. And one is shifted up to twice the carrier frequency.

Well it's this piece that we want. If we could eliminate everything else and keep this, then that would correspond to the spectrum of the original signal, x of t. How do we do that? Well, we know how to eliminate part of the spectrum and keep another part of the spectrum. That's called filtering. So what we would do is put the result of this through a low-pass filter. The low-pass filter route would retain the part of the spectrum around DC and eliminate the remaining part of the spectrum. So we would keep this part and eliminate the part of the spectrum that we have over here.

And let me just draw your attention to the fact that, because of the way the algebra works out, the amplitude of this replication of the spectrum is half what the original spectrum was. And that means that ideally, to keep scale factors correct, we would choose the amplitude of this to be 2, to scale this back up to 1.

So what we have is the modulator and demodulator. And just to summarize, for the case of a sinusoidal carrier as opposed to a complex exponential carrier, the modulator is just as it is in the complex exponential case. It's multiplication with the sinusoidal carrier, with frequency, omega c, and phase, theta sub c. In the demodulator we would take the modulated signal, modulate it again with the same carrier signal-- and as we'll see later, it's important to keep the same phase relationship.

This result is not yet quite the demodulated signal. We need to process that with a low-pass filter that extracts the part of the spectrum around DC and throws away the upper part of the spectrum that gets generated in the second modulation process. And the resulting output is the original signal, x of t.

What we've done then is we've taken x of t. We've modulated it onto a carrier. And then we've taken that modulated signal and we've figured out how to get back x of t. And of course one could ask, well, if you start with x of t and you want to get x of t back again, why bother going through all that? Why not just use x of t at the beginning and at the end? And obviously there are lots of reasons as I indicated before.

And just to reiterate what they are. The notion, often, is that what you'd like to do is shift the signal into a different frequency band for transmission over some medium that is more matched to that frequency band than the frequency range of the original signal. Also, as I alluded to, is the notion that you can take lots of signals and transmit them simultaneously over one channel-- whether the channel is a wire, a microwave link, a satellite link, or whatever-- again, using the idea of modulation. And what that process is referred to as is multiplexing.

And let me just quickly indicate what that multiplexing process corresponds to. We could think, for example, of taking one signal and modulating in it onto one carrier with one carrier frequency, taking a second signal, modulating it onto a different carrier frequency, taking a third signal and modulating it onto a third carrier frequency, et cetera. And if we choose these carrier frequencies appropriately, then we can add all those together-- and do it in such a way that the spectra don't overlap-- and end up with one broader band signal that incorporates the information simultaneously in all of those signals.

So just to illustrate that in the frequency domain. What we have are our three spectra, Xa, Xb, and Xc. And we would, for example, take this spectrum and modulate it to a carrier frequency, omega sub a. We can take this spectrum and modulate it to a carrier frequency, omega sub b, where omega sub b is chosen so that when we add these two together they don't overlap, so that they can eventually be separated out. And then we can do the same thing with the third signal, and put that in a frequency range over here, being careful that none of those overlap.

And when we add all those together, the composite spectrum is what I show here. And as you can see, essentially, by doing appropriate band-pass filtering we can pull out whatever part of the spectrum we choose to, and then demodulate that in the appropriate way. And of course we can do this, not just with three signals, but perhaps with tens or hundreds of signals.

So that's a process that is typically referred to as multiplexing. And as I've described it here, it's referred to as frequency-division multiplexing. That is, dividing the frequency band into cells and plunking different signals into each one of those.

And so if we want now to recover one of those channels in a frequency-division multiplex system, as I indicated, we would first demultiplex. Demultiplexing corresponding to pulling out the appropriate channel with a band-pass filter. And after demultiplexing, we would then demodulate. And we would demodulate with the carrier appropriate to the channel that we've pulled out. And the demodulation, of course, involves multiplying by the carrier and doing appropriate low-pass filtering to finally get the signal back.

And frequency-division multiplexing is the type of multiplexing that's used, for example, in typical broadcast AM radio systems, where all the channels are superimposed together. And it's your home radio receiver that does the appropriate demultiplexing and demodulating. And of course, you can see that not only is modulation an important part of that, but as I alluded to in the last lecture, filtering also becomes important part of these practical systems.

Now, the kind of amplitude modulation that I've talked about so far is what's referred to as synchronous modulation. And the reason for the term synchronous is that what's implied in these systems is a synchronization between the transmitter and receiver. In particular, in the system as we've talked about it, the modulator and the demodulator have a synchronization in both frequency and phase. The phase here is indicated as theta sub c. And if we take a look at the demodulator, the demodulator has phase of theta sub c.

And in general, there's the issue of whether we can maintain that synchronization between the modulator and demodulator. And so what we want to examine now, more generally, is what the consequence might be, and the solution to the resulting problems, if we don't have synchronization between the modulator and demodulator. Synchronization in terms of phase. And there also is another problem, which is the issue of synchronization in frequency. That's examined more in the text. And what I'll focus on here is just the issue of synchronization in phase, to give you some sense of what the issue is.

So now what we want to look at is what happens if we have a modulator with phase, theta sub c, and a demodulator where the phase, instead of being theta sub c, is some other phase, phi sub c. And if you track through the details and the algebra, then what you'll find is that the output of the low-pass filter, rather than being x of t, the signal that we want, is x of t multiplied by a scale factor. And the scale factor is the cosine of the phase difference.

Now one could ask, OK well, what's the big deal about scale factor? If it's too small we'll make it big, it it's too big we'll make it small. But there are several points. One is, notice, for example, that if the phase difference between the modulator and demodulator is 90 degrees, then the output of the demodulator is zero. Or if it isn't quite 90 degrees, the amplitude might be small. And the implication would be that if there's other noise it gets injected in the system, the signal-to-noise ratio is very low.

Now even worse is the issue that if there's a phase difference, but the exact phase difference isn't maintained, so that the modulator and demodulator kind of fade in and out of phase, then the output of the demodulator is x of t multiplied by a time-varying fading term, which is the cosine of the phase difference. Well what that means, essentially, is that if you use this kind of system to do the demodulation, then what you need to be careful about is maintaining synchronization in phase, and also in frequency, between the modulator and the demodulator.

Now there are alternatives to this. And the alternative is what's referred to as asynchronous demodulation. And let me indicate what the idea behind asynchronous demodulation is.

Now, recall that what we've done in amplitude modulation is to take the carrier signal and vary its amplitude with the signal that eventually we want to get back. So if we look at the amplitude-modulated waveform, it might typically look as I indicate here. And we're trying to get back the envelope.

Well, one could imagine building a circuit, or designing a device, which in some sense will track the envelope. And a common circuit to do that is a fairly simple circuit consisting of a diode and a resistor and capacitor in parallel. The idea being that the capacitor charges up as this waveform moves up to its peak. And then as the waveform drops down, the capacitor discharges through the resistor. And it kind of tracks the envelope. In fact, the kind of output that we would get is the type of behavior that I've indicated here.

And then that is a type of demodulation. It's a demodulation that doesn't require synchronization between the modulator and demodulator. And it's fairly inexpensive to build. But it has, obviously, some tradeoffs associated with it.

Well, to indicate where the tradeoff comes from, or where the issue surfaces, notice that what we're doing is tracking the envelope of the sinusoidal signal. And we're calling that, or we're assuming that that is our original signal, x of t. Well, suppose that x of t, the original signal, is sometimes positive and sometimes negative. What might we see as we look at the output of the demodulator? Well, the output of the demodulator would follow the envelope down, and then it would follow the envelope back up again. In other words, what it would tend to generate is a full-wave rectified version of the signal that you were really trying to get back.

Now, there's a simple solution to this. The simple solution is to make sure that the signal that is the modulating signal, x of t, never goes negative. So if it happens to-- a voice signal tends to go negative. If it happens to, we can simply add a constant to it, add a large enough constant, so that it always stays positive.

Well let's look at that. What we want to do then, if we're considering asynchronous demodulation, is to take our original signal, x of t, and add to it a constant, where the constant is made large enough so that we're sure that this is a positive signal. And incidentally, let me just draw your attention to the fact that I'm now suppressing the phase on the carrier signal, since the phase is not important to the argument and it's just some additional notation to carry around.

So the idea then, is add a constant to x of t. Notice that if we just take this term and expand it out into two terms, x of t cosine omega c t plus a times cosine omega c t, then in block diagram terms we can represent that as I've shown here. And so it would correspond to modulating the signal, x of t, onto the carrier, omega sub c t, and also injecting some carrier with an amplitude, A. And the output of the modulator is then the sum of those two.

And depending on exactly what this value A is will influence what the envelope will look like. And I indicate below, two possibilities. One is where I've made A fairly large, and one is where I've made A significantly smaller. And there are both positive and negative issues associated with whether A is too large or A is too small. For example, if A is large in relation to the amplitude of the signal, then this envelope tends to be very flat. And it tends to be easy to track it with that simple diode RC circuit, as compared with the case down here.

On the other hand, there is a price that you pay for this kind of envelope. And the price that you pay is perhaps best seen in the frequency domain. If we look in the frequency domain, here is our original spectrum. Here is the spectrum at the output of the modulator. And the impulse that occurs here corresponds to the carrier that's injected. The larger A is, the more carrier is injected. The more carrier that's injected, the easier it is for the envelope detector to demodulate.

So one can ask, why not just put a lot in? Well, the obvious answer is that it's not an information-carrying part of the signal. And so in some sense it represents an inefficiency in transmission, because what you're transmitting is power, energy, that doesn't have any information associated with it. It's simply the injection of a carrier to make the demodulation for an asynchronous demodulator-- to make the demodulation easier.

And so there's this tradeoff. And in fact, one represents the tradeoff and the associated parameters very often in terms of percent modulation, where the percent modulation is essentially the ratio of the maximum signal level to the amplitude of the injected carrier. And depending on whether the modulation's very high or very low, the tradeoff is that the transmission is more inefficient and it takes more energy, but the demodulator is simpler. Or the demodulator is more complicated but the transmission is simpler.

Now, there are situations where you might very well want to use one or the other. For example, in home radio you're often willing to transmit a lot of power so that you can have inexpensive consumer-oriented receivers. On the other hand, in satellite communication you're willing to pay a very high price for the modulators and demodulators, but it's the amount of power that's transmitted that's at a premium. And so in one case, satellite communication, you would use synchronous modulation and demodulation. Whereas in typical consumer-oriented broadcasting, you would use an asynchronous system and transmit more power, even if it's inefficient, so that the demodulator can be simpler.

Now, in the asynchronous system, as we've indicated, there's one source of inefficiency, which is this injection of the carrier. There also is a somewhat different issue, related to inefficiency in sinusoidal amplitude modulation. And it's an inefficiency that is separate from the issue of synchronous versus asynchronous systems. In other words, it's not associated with the injection of the carrier, it's a very different issue. Let me indicate what that is.

Let's look again at the spectrum of x of t, which I've indicated here. And in a sinusoidal amplitude modulation system, we would center it around plus and minus the carrier frequency. Now, notice that in the original system we occupy a frequency spectrum that's 2 times omega sub M. By the time we've shifted it, thinking of positive and negative frequencies, we've used up twice as much of the frequency spectrum.

Well you could say, OK, let's just shift this up this way and get rid of this part. That's of course what the complex exponential carrier did. And the issue there is that now you've got to transmit both a real part and an imaginary part.

So what you can think about, and ask, is if you still want to transmit a real-valued signal, how can you somehow remove the inefficiency or redundancy in the spectrum? Well, notice that what we have is this spectrum moved here, and moved here. And we could imagine building real-valued signal by eliminating what I refer to here as the lower sideband out of the positive frequencies, and the lower sideband out of the negative frequencies. And in effect, what we've done is taken just the positive frequencies here, shifted them there, the negative frequencies here, and shifted them here. And the resulting spectrum is what I indicate below.

Well, this is what is often done. And what it's referred to as is single sideband. What we've done is kept the upper sideband, in this particular case. We could alternatively think of putting this system together where we retain the lower sideband instead of the upper sideband. And in either case, what we've removed is an inefficiency in transmission of the signal. Namely we have a real-valued signal, but it only requires as much total bandwidth, in terms of the frequencies in which there's energy present, as the original signal.

Well how do we do this? There are a variety of ways. And there's one procedure that is discussed in more detail in the text, which uses what's referred to as a 90 degree phase splitter. The simplest way, at least conceptually, is to think about doing it with filtering.

And the idea simply is that if we have our modulated signal-- here's the spectrum of the modulated signal. And if that modulated signal is simply put through a high-pass filter, then the result will be to eliminate the lower sideband, if we choose the high-pass filter to have a characteristic as I indicate here. So this, conceptually, is a very sharp cutoff filter. And what it eliminates are the lower sidebands. And the resulting spectrum is what we have below.

And this in fact is really the basic idea behind single-sideband transmission. Again, there's a tradeoff. It's clearly more efficient than double-sideband transmission, but also has the complication, or additional issue, that the modulator becomes a little more complicated because you need this filtering operation, or some equivalent operation, to get rid of the unwanted sideband.

Well, this is a fairly quick tour through a variety of issues related to modulation. And it really is just the tip of the iceberg, obviously. Modulation in the context of sinusoidal modulation, as we've talked about, has a lot of detailed issues associated with it.

It's important to recognize, and to be somewhat pleased by the fact, that not only with the mathematical foundations that we've developed can we understand the basics of sinusoidal amplitude modulation. But what you'll find if you dig into this somewhat deeper that the basic background that we built up so far-- the mathematical tools-- are really pretty much what you need for a much deeper understanding of all of the issues involved. So from what might have seemed like a fairly abstract mathematical property associated with the Fourier transform, we've begun to develop what should give you the sense of some important practical considerations.

And as we'll see the next lecture, very much the same kinds of notions apply for discrete time, sinusoidal, and complex exponential amplitude modulation. And also as I indicated at the beginning of the lecture, in the next lecture we'll also talk about what's referred to as pulse amplitude modulation. It's a different kind of carrier. And what that will lead to, among other things, is a very important bridge between the notions of continuous time and the notions of discrete time. Thank you.