NETWORK STRUCTURES FOR IIR SYSTEMS

Solution 12.1

(a)   Direct Form I (text figure 6.10) corresponds to first implementing
the right-hand side of the difference equation (i.e. the zeros)
followed by the left-hand side (i.e. the poles).  Thus the direct form
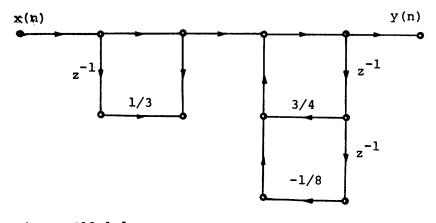I for this difference equation is:



Figure S12.1-1

The flow-graph drawn in this form graphically separates the part
implementing the zeros and that implementing the poles.  It can
alternatively be drawn more efficiently by eliminating some of the
branches with unity gain and collapsing some of the nodes.  For
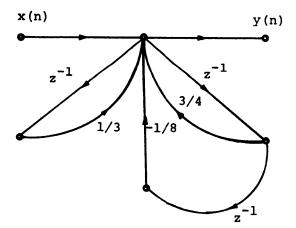example an alternative way of depicting the flow-graph of Figure
S12.1-1 is:



Figure S12.1-2

(b)   The direct-form II (text Figure 6.11) corresponds to implementing
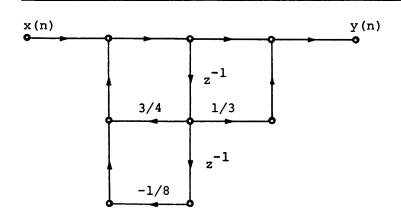the poles first, followed by the zeros:
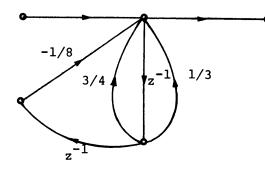
Figure S12.1-3

or alternatively,



Figure S12.1-4

(c)   In the cascade form using first-order sections, we must first factor the system function into a cascade of two first-order systems. Applying the z-transform to both sides of the difference equation,

$$Y(z) \left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right] = X(z) \left[1 + \frac{1}{3}z^{-1}\right]$$

or

$$H(z) = \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{1 + \frac{1}{3}z^{-1}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{2}z^{-1})}$$

In developing the cascade form, we can include the zero with either pole and arrange the cascade in either order.   For example writing H(z) as

$$H(z) = \left[\frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{1}{4}z^{-1}}\right] \left[\frac{1}{1 - \frac{1}{2}z^{-1}}\right]$$

and using the direct-form II for the first subsection leads to the
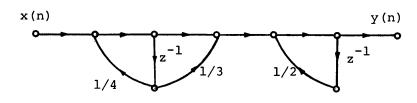
cascade form shown in Figure S12.1-5



Figure S12.1-5

This flow-graph can also be collapsed somewhat as we have done with those in (a) and (b).

(d)   The parallel form corresponds to expanding H(z) in a partial fraction expansion.  Thus,

$$H(z) = \frac{10/3}{1 - \frac{1}{2}z^{-1}} + \frac{-7/3}{1 - \frac{1}{4}z^{-1}}$$
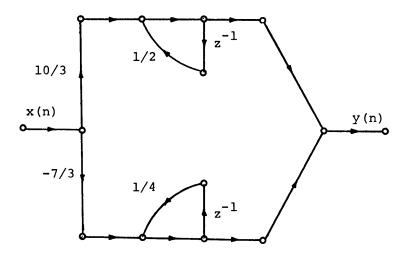
leading to the flow graph shown below:



Figure S12.1-6

Solution 12.2

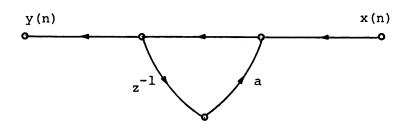(a)   The transpose of this network is:



Figure S12.2-1

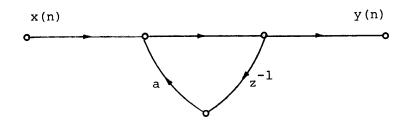or, drawing it with the input on the left and the output on the right,



Figure 12.2-2

Since the only effect of the transposition is to interchange the order of the delay and coefficient branches in the feedback path the transfer function is clearly unchanged.

(b)  Having worked problem 12.1 we can write down the transfer function of this network by inspection:

$$H(z) = \frac{1 + \frac{1}{4}z^{-1}}{1 - \frac{1}{2}z^{-1}}$$
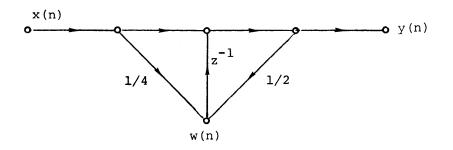
The transposed network is



Figure S12.2-3

It is not obvious by inspection (at least not to me) that this transposed network has the same transfer function as the original. However writing the equations for the above flow graph we obtain:

$$w(z) = \frac{1}{2} Y(z) + \frac{1}{4} X(z)$$

and

$$Y(z) = X(z) + z^{-1} W(z)$$

Combining these two equations, we obtain

$$Y(z) = X(z) + z^{-1} \left[ \frac{1}{2} Y(z) + \frac{1}{4} X(z) \right]$$

$$\frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{4}z^{-1}}{1 - \frac{1}{2}z^{-1}}$$

(c)  By inspection of the network, we can write that

$$Y(z) = aX(z) + bz^{-1} X(z) + cz^{-2} X(z)$$

or

$$H(z) = a + bz^{-1} + cz^{-2}$$
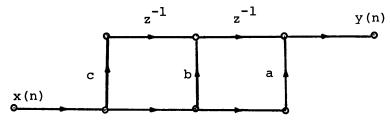
The transpose network is



Figure S12.2-4

By inspection of this network we see that

$$Y(z) = cz^{-2} X(z) + bz^{-1} X(z) + aX(z)$$

or $H(z) = a + bz^{-1} + cz^{-2}$
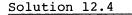
as before.

## Solution 12.3

By inspection we observe that for network (a) there is a delay free path
with a gain of 2 from input to output, i.e. if $x(n) = \delta(n)$ then
$y(0) = 2$.  Since this is not true for networks (b) or (c), they are
eliminated.  Second, by inspection of (a) we note that there are two
poles, one at $z = 1/2$ and one at $z = -3/4$, i.e. the denominator of
the system function is

$$(1 - \frac{1}{2}z^{-1})(1 + \frac{3}{4}z^{-1}) = (1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}).$$

Since networks (e) and (f) correspond to system functions with
denominator polynomial given by

$$(1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2})$$

these networks are eliminated. The remaining possibility, which is
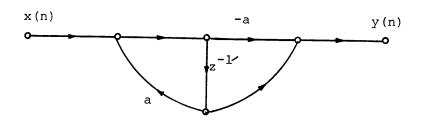the correct answer, is network (d).

## Solution 12.4

(a)



Figure S12.4-1

Note that one delay and two multipliers are required.

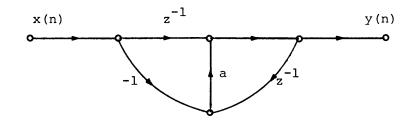(b)

$$y(n) = a\left(y(n-1) - x(n)\right) + x(n-1)$$



Figure S12.4-2

In this case two delays but only one multiplier are required.

(c)   First, let us simply cascade two networks of the form obtained
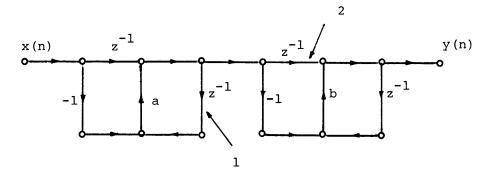in (b):



Figure S12.4-3

Now, we note that the input and output of the two branches labelled

"1" and "2" are identical and thus these can be combined into a single delay. For depicting the final network it is also convenient to "flip" the second network above.
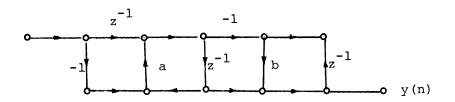


Figure S12.4-4

## Solution 12.5

Since each section is 3.4 cm. long and the velocity of sound is $3.4 \times 10^4$ cm/sec. it takes $10^{-4}$ secs. to traverse one section. With a sampling rate of 20 khz the sampling interval is $.5 \times 10^{-4}$ seconds and consequently the length of each section is two sampling intervals, i.e. each can be represented by two delays in cascade. The entire system is linear and so the forward and backward travelling waves add at the boundaries. Let

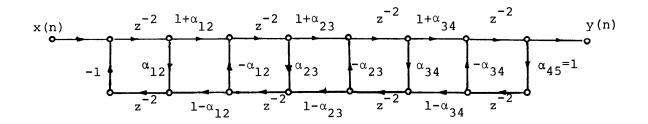$$\alpha_{kn} = \frac{(A_n - A_k)}{(A_n + A_k)} \quad .$$

Then the resulting network is



Figure S12.5-1

Resource: Digital Signal Processing
Prof. Alan V. Oppenheim