S O L U T I O N S

Conditional Stability 17

Note: All references to Figures and Equations whose numbers are *not* preceded by an "S" refer to the textbook.

Solution 17.1 (P6.9)

The limiter characteristics are shown in the first entry of Table 6.1. For the purposes of this problem, let $K = E_M = 1$. Then, from Table 6.1,

$$G_D(E) = 1 \measuredangle 0° \quad E < 1 \tag{S17.1a}$$

$$G_D(E) = \frac{2}{\pi}(\sin^{-1}R + R\sqrt{1 - R^2}) \measuredangle 0° \quad E > 1 \tag{S17.1b}$$

where $R = \dfrac{1}{E}$.

We want to combine this nonlinearity with a transfer function $a(s)$ in a loop of the form shown in Figure 6.9. Given this topology, and the $G_D(E)$ in Equation S17.1, a gain-phase plot of $-\dfrac{1}{G_D(E)}$ and $a(j\omega)$ that will yield stable amplitude limit cycles at two different frequencies is shown in Figure S17.1. The two intersections with positive slope of the $a(j\omega)$ curve represent stable oscillation points, as shown.

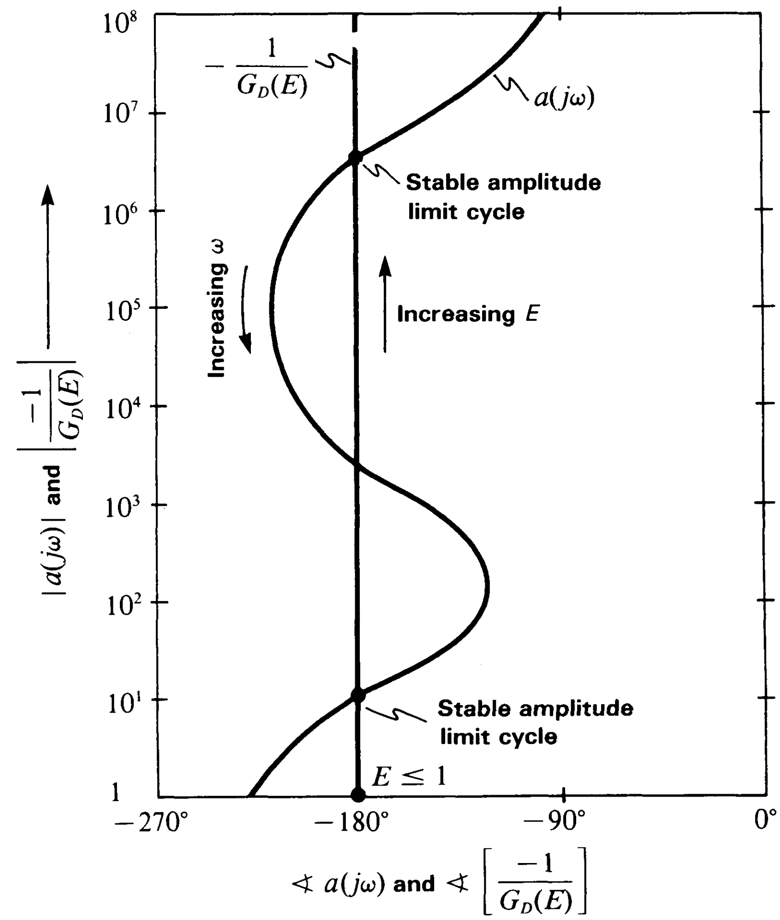
An $a(j\omega)$ that realizes the curve indicated in Figure S17.1 is

$$a(s) = \frac{K(0.05s + 1)^2}{s(s + 1)^2(10^{-3}s + 1)^2} \tag{S17.2}$$

There are many possible $a(j\omega)$ that will result in two stable oscillation points, and the $a(j\omega)$ indicated in Equation S17.2 is just one of them. They all have the common characteristic of two distinct regions of the $a(j\omega)$ curve where $\measuredangle\, a(j\omega) < -180°$. These regions are separated by a region where $\measuredangle\, a(j\omega) > -180°$. This is one of those problems where there will be as many different solutions as there are students.

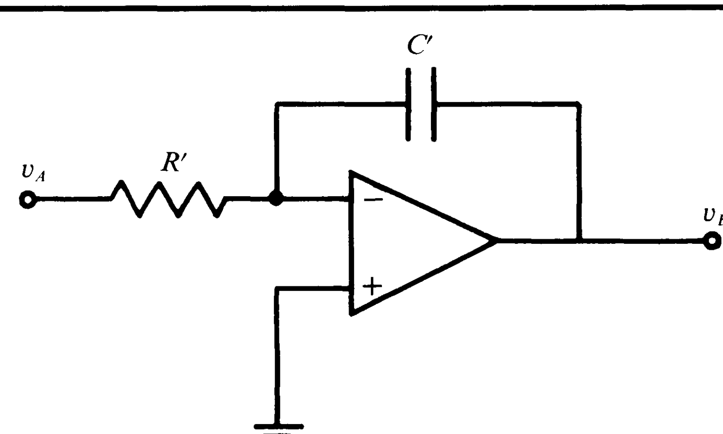
Given the $a(s)$ of Equation S17.2, we notice that it combines an integrator, two identical lag networks with $\alpha = 20$ formed by the poles at $s = -1$ and the zeros at $s = -20$, and a pair of poles at $s = -10^3$. The constant K is used to adjust the amplitude of the oscillations.

Figure S17.1 Describing function analysis for system of Problem 17.1 (P6.9) with two stable amplitude limit cycles.



The circuit shown in Figure S17.2 realizes the integration and provides a gain constant $G = \dfrac{-1}{R'C'}$. The negative sign of G supplies the inversion indicated in Figure 6.9.

Figure S17.2 Integration for Problem 17.1 (P6.9).



Using the results of Section 5.2.3, we can design a lag network with its pole at $s = -1$ and its zero at $s = -20$. That is, $\alpha\tau = 1$, and $\tau = 0.05$, $\alpha = 20$. This network appears as shown in Figure S17.3.

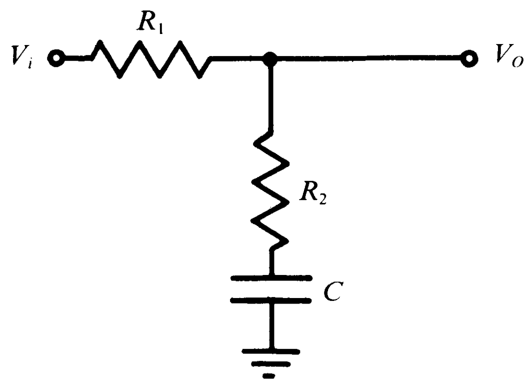


Figure S17.3 Lag network for Problem 17.1 (P6.9).

Now, if we cascade two such networks, and arrange to have the input impedance of the second network much larger than the output impedance of the first network, loading will be insignificant. Thus, for the first network, because $\alpha = \dfrac{R_1 + R_2}{R_2}$ and $\tau = R_2C$, let $R_1 = 100$ kΩ. Then to set $\alpha = 20$, $R_2 = 5.26$ kΩ. Then, because $\tau = 0.05$, we have $C = 9.5$ μF. For the second network, to reduce loading, we multiply the resistances of the first network by 20 and divide the capacitance by 20, to maintain the same α and τ. (An operational amplifier acting as a buffer between the two sections could be used to reduce loading further.) With this impedance scaling, the second lag network uses $R_1 = 2$ MΩ, $R_2 = 105$ kΩ, and $C = 0.48$ μF. The cascaded lag networks form the center section of the circuit shown in Figure S17.4.

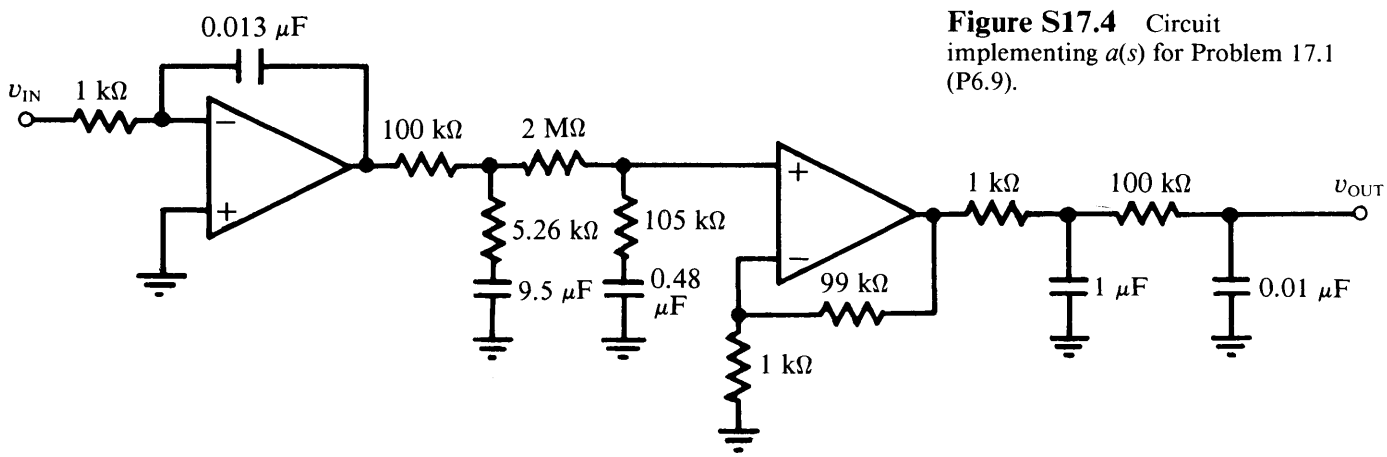


Figure S17.4 Circuit implementing $a(s)$ for Problem 17.1 (P6.9).

We follow the lag networks with a gain of 100 connected operational amplifier to act as a buffer and provide gain. Finally, the two poles at $s = -10^3$ must be implemented. As shown in Figure S17.4, we choose to implement these poles with the cascade of two first-order sections with $RC = 10^{-3}$. Again, the impedance of the second section is scaled up relative to the first section (in this case by a factor of 100) to minimize loading, and make our calculations easier.

Finally, the integrator gain constant G must be adjusted to ensure that the $|a(j\omega)| > 1$ when $\measuredangle a(j\omega)$ crosses through $-180°$ for the last time. Calculations show that this occurs for $\omega = 10^3$ rad/sec, and if G is chosen as $G = -8 \times 10^4$ (recall that we also have a gain of 100 from the second amp, thus $K = 100\ G$), this crossing will occur with $|a(j\omega)| = 10$. This ensures an intersection with the $-\dfrac{1}{G_D(E)}$ curve, which does not continue below $\left|\dfrac{1}{G_D(E)}\right| = 1$. Then, because $G = \dfrac{-1}{R'C'}$, we have $R'C' = 1.25 \times 10^{-5}$, which can be satisfied by $R' = 1\ \mathrm{k}\Omega$, and $C' = 0.013\ \mu\mathrm{F}$. The complete circuit is shown in Figure S17.4.

MIT OpenCourseWare
http://ocw.mit.edu

RES.6-010 Electronic Feedback Systems
Spring 2013

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.