In this segment, we will discuss what a sequence is and what it means for a sequence to converge.

So a sequence is nothing but some collection of elements that are coming out of some set, and that collection of elements is indexed by the natural numbers.

We often use the notation, and we say that we have a sequence ai, or sometimes we use the notation that we have a sequence of this kind to emphasize the fact that it's a sequence and not just a single number.

And what we mean by this is that we have i, an index that runs over the natural numbers, which is the set of positive integers, and each ai is an element of some set.

In many cases, the set is going to be just the real line, in which case we're dealing with a sequence of real numbers.

But it is also possible that the set over which our sequence takes values is Euclidean space n-dimensional space, in which case we're dealing with a sequence of vectors.

But it also could be any other kind of set.

Now, the definition that I gave you may still be a little vague.

You may wonder how a mathematician would define formally a sequence.

Formally, what a sequence is, is just a function that, to any natural number, associates an element of S. In particular, if we evaluate the function f at some argument i, this gives us the ith element of the sequence.

So that's what a sequence is.

Now, about sequences, we typically care whether a sequence converges to some number a, and we often use this notation.

But to make it more precise, you also add this notation here.

And we read this as saying that as i converges to infinity, the sequence ai converges to a certain number a.

A more formal mathematical notation would be the limit as i goes to infinity of ai is equal to a certain number, a.

But what exactly does this mean?

What does it mean for a sequence to converge?

What is the formal definition?

It is as follows.

Let us plot the sequence as a function of i.

So this is the i-axis, and here we plot entries of ai.

For a sequence to converge to a certain number a, we need to the following to happen.

If we draw a small band around that number a, what we want is that the elements of the sequence, as i increases, eventually get inside this band and stay inside that band forever.

Now, let us turn this into a more precise statement.

What we mean is the following.

If I give you some positive number epsilon, and I'm going to use that positive number epsilon to define a band around the number a.

So it's this band here.

If I give you a positive number epsilon, and therefore, this way, have defined a certain band, there exists a time after which the entries will get the inside the band.

In this picture, it would be this time.

So there exists a time-- let's call that time i0-- so i0 is here such that after that time, what we have is that the element of the sequence is within epsilon of a.

So this is the formal definition of convergence of a sequence to a certain number a.

The definition may look formidable and difficult to parse, but what it says in plain English is pretty simple.

No matter what kind of band I take around my limit a, eventually, the sequence will be inside this band and will stay inside there.

Convergence of sequences has some very nice properties that you're probably familiar with.

For example, if I tell you that a certain sequence converges to a number a and another sequence converges to a number b, then we will have that ai plus bi, which is a new sequence-- the ith element of the sequence is this sum-

- will converge to a plus b.

Or similarly, ai times bi, which is another sequence, converges to a times b.

And if, in addition, g is a continuous function, then g of ai will converge to g of a.

So for example, if the ais converge to a, then the sequence ai squared is going to converge to a squared.