As before, we have a red Poisson process and a green Poisson process.

We merge these two processes, and we only observe the merged process.

Here's an interesting question.

This is an arrival of the merged process.

Where did it come from?

Is it red, or is it green?

We cannot know, but can we tell what is the probability that it came from the red stream?

The way to answer this question is to look at the table of all the things that can happen during a little interval around that particular time in which we had an arrival.

We are told that there was an arrival at time t or an arrival during an interval, a small interval around time t.

This means that we're told that this event here has happened.

Given this information, what is the conditional probability that actually this event here occurred?

This is just the fraction of this probability divided by the total probability of the conditioning event.

So the answer is lambda 1 divided by lambda 1 plus lambda 2.

Does this answer make sense?

Well, suppose that lambda 1 and lambda 2 were equal.

In that case, by symmetry, when an arrival comes, it should be equally likely to have come either from the red or from the green stream.

And this is consistent with this answer.

We can reason similarly for a slightly different question.

You wait until the kth arrival, let's say the third arrival.

Where did that arrival come from?

Well, that case, arrival occurred during a specific little time interval and conditioning on it having occurred during that particular time interval, we can then repeat the reasoning that we had here and argue that given that we had an arrival-- it just happens to be the third arrival during that time interval-- there's going to be this particular conditional probability that it came from the red stream.

So we obtained the same answer once more.

Now, this arrival came from one of the two streams with some probabilities.

This arrival came from one of the two streams with some probabilities.

Does the origin of this arrival affect or depend on the origin of that arrival?

Because we have assumed independence across time for each one of the processes that we started with-- and therefore, we also have the same thing for the merged process-- whatever has to do with events during this interval is independent from anything that has to do with events in that interval.

And because of this, one could argue formally-- but hopefully, this is intuitive enough-- that the origin of this arrival and the origin of that arrival are independent events.

And now that we have this property, we can answer questions such as the following.

We've had 10 arrivals so far.

What is the probability that exactly four out of these 10 are red?

Each one of those arrivals has this probability of being red.

The origin of different arrivals are independent of each other.

So essentially, we're dealing with 10 Bernoulli trials, each of which has two possible outcomes, red or green, and is red with this particular probability.

Therefore, the answer is going to be given by the binomial probabilities, which is the probability of having four successes in 10 trials.

And we obtain lambda 1 over lambda 1 plus lambda 2.

That's the probability of a red to the number of red arrivals.

And then the remaining probability, 1 minus that, which is lambda 2 over lambda 1 plus lambda 2 to the remaining power, which is equal to 6.

So to summarize.

Each one of the arrivals in the merged process has a certain probability of being a red arrival or a green arrival.

Which one of the two is the case?

We can think of it as an outcome of Bernoulli trial, and the Bernoulli trials associated with different arrivals are independent of each other as a consequence of the independence of Poisson processes across time.