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PHYS 201
RESOURCES

# Flux and Gauss' Law

## Derivatives and Integrals Series

## *Instructor's Guide*

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DEVELOPED BY THE TEACHING AND LEARNING LABORATORY AT MIT
FOR THE SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

# Introduction

## Key Information

*Duration:* 17:57
*Narrator:* Prof. Peter Fisher
*Materials Needed:*
- Paper
- Pencil/pen

## When to Use this Video

- In Phys 201, during Lecture 6 or 7, or an associated discussion section. This video is also suitable to assign for homework, either after Lecture 5 or as review for Exam 1.
- Students must be familiar with Gauss' Law at a basic level, and should have either seen or calculated for themselves some of the more typical cases. They should also know how to take an area integral.

## Learning Objectives

After watching this video students will be able to:

- Implement Gauss' Law more effectively.
- Give an organized view of electric flux.
- Categorize situations by symmetry.
- Learn how to choose a Gaussian surface.

## Motivation

- Existing videos on Gauss' Law are typically either extremely math-oriented or highly conceptual. This video seeks a balance between the two.
- Most existing Gauss' Law videos are focused on an introduction to the law.

## Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Take notes.
- Choose appropriate Gaussian surfaces for use in a set of Gauss' Law problems.

## Video Highlights

This table outlines a collection of activities and important ideas from the video.

| Time | Feature | Comments |
|---|---|---|
| 1:30 | Review of flux equations | |
| 1:47 | Using symmetry | This segment reviews the factors that allow us to remove variables from an integral, particularly, the electric flux integral. |
| 5:12 | Charge distributions | The discussion of symmetry continues here. |
| 8:05 | Approximations | There is no mathematical discussion of the approximations, only a description of where they are appropriate. |
| 9:12 | Picking a surface | This segment begins with a review. Students are then asked to determine the best shape for a Gaussian surface in a particular situation. |
| 14:08 | Review | |
| 14:30 | What is flux? | This video clip shows professors and graduate students from 15:52 to 18:38. |

## Video Summary

This video examines Gauss' Law and area integrals in detail, focusing on extracting as many variables as possible from the integral through careful choice of a Gaussian surface. Symmetry is defined and used extensively, and students have an opportunity to choose a surface that will maximize symmetry for a given situation. The video also includes a brief discussion of when approximations are appropriate, and finishes with MIT professors and postdoctoral students explaining how they think about flux.

# Phys 101 Materials

## Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.

**1.** Flux Cubes (Appendix A1-3)

This is a set of three clicker questions involving point charges near a cube. The first is a straightforward application of Gauss' Law that should be used to gauge whether students need more work on the basics of the law. The second and third require increasing amounts of three-dimensional thinking and visualization.

If students have difficulty in the second problem with whether the point charge is inside or outside of the cube, it may be useful to have them instead consider a very small sphere with its center at the corner of the cube, such that one eighth of it is inside the cube.

**2.** Flux Through Surface (Appendix A2-1 and A2-2)

These questions are similar to those in the previous set, except that now the charge is off-screen and the field takes center stage. Three-dimensional visualization is again important. There is also the question of whether one cares about the *least positive* flux or the *lowest magnitude* of flux (and thus can always choose two answers rather than just one).

The second question in this set has a certain amount of intentional ambiguity and arguability to it. If students cannot agree as to whether (for example) the front or right sides have the least positive flux, ask them what information they want in order to answer the question definitively.

The dots drawn on the cube are to show where the field lines exit its surface; they are not charges.

**3.** Flux and Circles (Appendix A3)

The intent of this question is to improve students' instinctual feel for field line shapes and the presence or absence of electric charge. If students have difficulty with this problem, ask them to identify where the charges are in each diagram and whether or not they are enclosed within the surfaces.

Item #6 has some ambiguity, depending on the exact charge of the two locations.

## Post-Video Materials

**1.** Electric Field Worksheets (Appendix A4 and A5)

A pair of worksheets are included in the appendix, asking students to calculate the electric field in regions near a cylindrically symmetric charge distribution and a spherically symmetric distribution. Students are asked to find the field in *all* regions, meaning at any distance from the center of the charge distribution (not just between the objects).

The problems have a great deal of similarity in their appearance and method of solution. It is important to note that the cylindrical situation involves charged conductors, but the spherical situation involves one conductor and one insulator.

These problems can be handed out as worksheets or displayed on a screen during class or a recitation section.

# Additional Resources

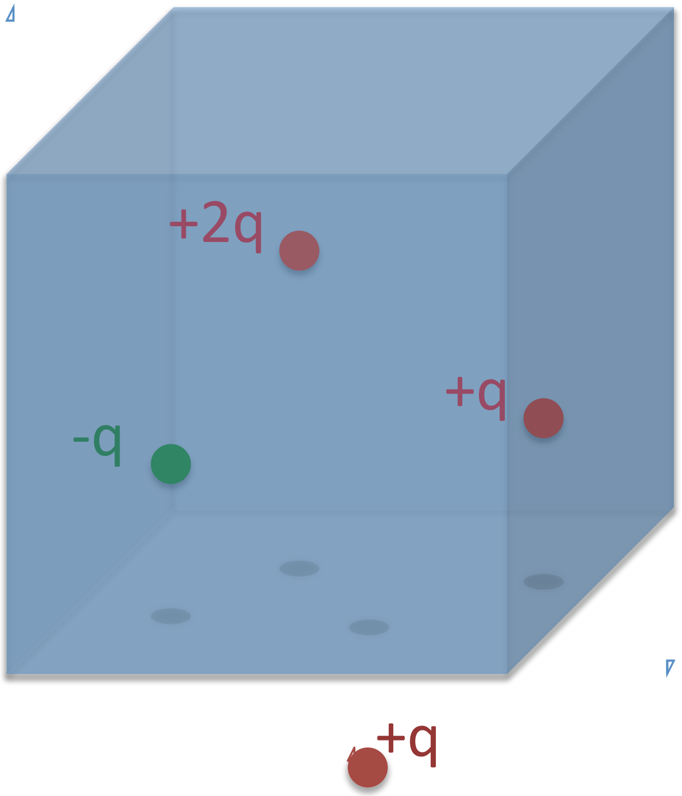
## Going Further

The Going Further folder contains a Maple worksheet involving a situation with less than ideal symmetry, to show a different approach for finding the field. This worksheet integrates the charge distribution to find the electric potential, and applies a gradient to find the field. The worksheet uses a few sophisticated mathematical functions (the Dirac Delta and the Heaviside step function), but should be understandable to students who are familiar with or who are willing to investigate those functions.

## References

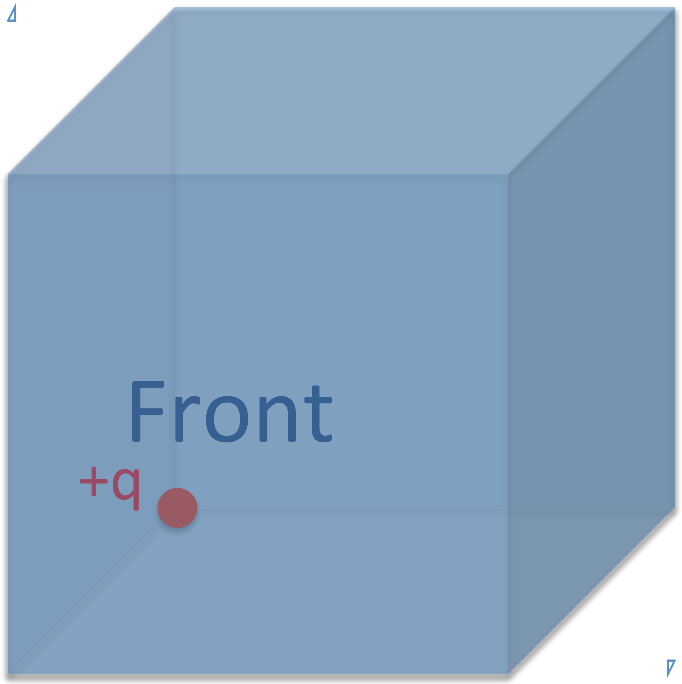
Gauss' Law's connection to vectors, multivariable calculus, and electromagnetism all make it a rich source of difficulty for students. A sampling of research on student difficulties and possible ways to help are listed below. A reference to Ampere's Law is also included, as it makes explicit connections between Amperian loops and Gaussian surfaces.

- Sing, C. (2006). Student understanding of symmetry and Gauss's law of electricity. *American Journal of Physics 74*(10), 923-936.
- Planinic, M. (2006). Assessment of difficulties of some conceptual areas from electricity and magnetism using the Conceptual Survey of Electricity and Magnetism. *American Journal of Physics 74*(12), 1143-1148.
- Chabay, R., & Sherwood, B. (2005). Restructuring the introductory electricity and magnetism course. *American Journal of Physics 74*(4), 329-336.
- Dori, Y., & Belcher, J. (2005). How Does Technology-Enabled Active Learning Affect Undergraduate Students' Understanding of Electromagnetism Concepts? *The Journal of the Learning Sciences, 14*(2), 243–279.
- Manogue, C., Kerry Browne, K., Dray, T., Edwards, B. (2006). Why is Ampère's law so hard? A look at middle-division physics. *American Journal of Physics, 74*(4), 344-350.



Several charges sit inside or near a cubical object as shown. What is the value of the net flux through the object?

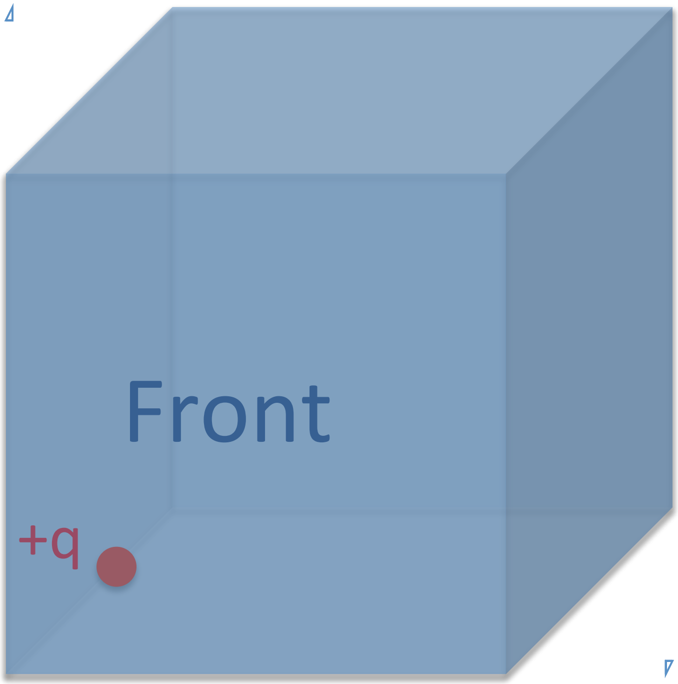
① $5q/\varepsilon_0$
② $4q/\varepsilon_0$
③ $2q/\varepsilon_0$
④ Zero
⑤ $-2q/\varepsilon_0$
⑥ Impossible to tell



A small spherical charge +q sits with its center at the back, bottom, left-hand corner of a cube.

Which sides of the cube have the greatest electric flux through them?

① The front
② The back
③ The right-hand side
④ The left-hand side
⑤ The top
⑥ The bottom
⑦ There is no electric flux through any face of the cube.



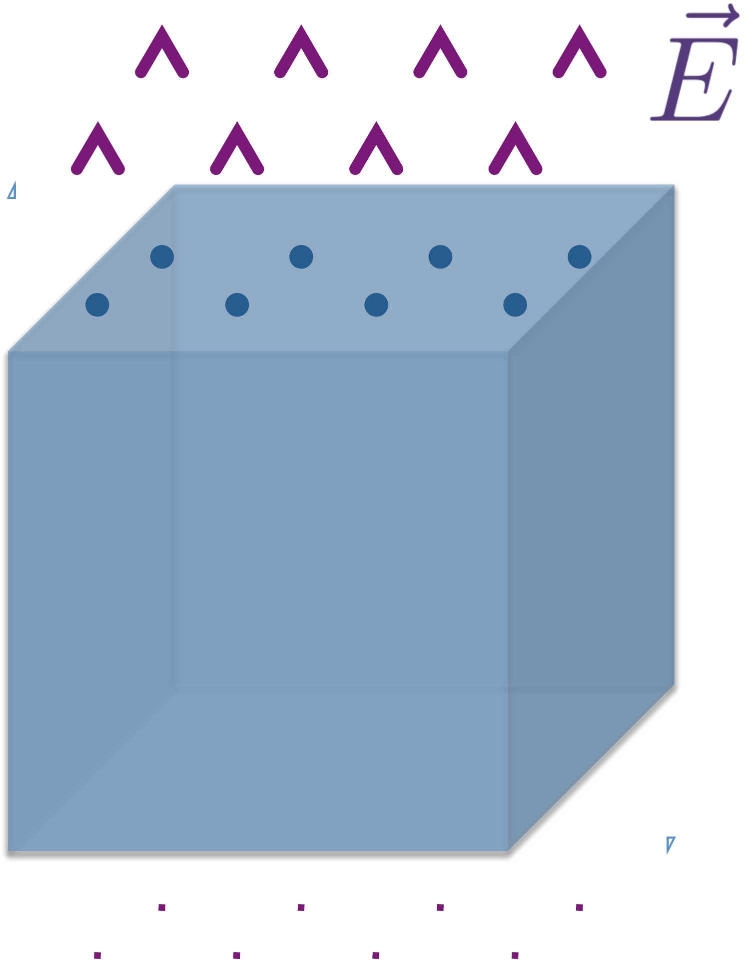
A small spherical charge +q sits with its center in the middle of the bottom left-hand edge of a cube.

Which sides of the cube have the greatest electric flux through them?

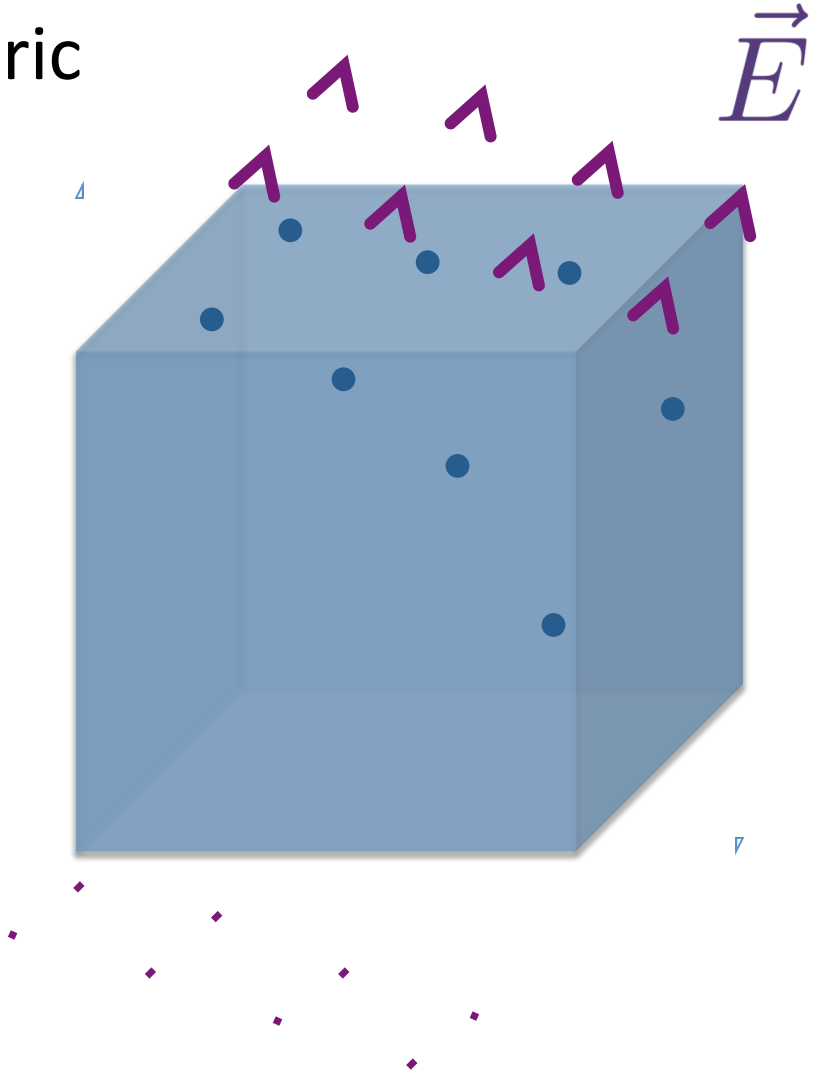
① The front
② The back
③ The right-hand side
④ The left-hand side
⑤ The top
⑥ The bottom
⑦ There is no electric flux through any face of the cube.

A cube sits in a uniform electric field as shown. Which sides of the cube have the greatest electric flux through them?

① The front
② The back
③ The right-hand side
④ The left-hand side
⑤ The top
⑥ The bottom
⑦ There is no electric flux through any face of the cube.
⑧ Impossible to tell

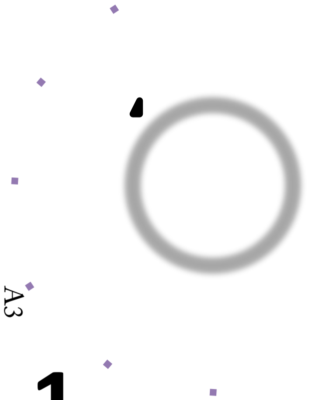


A cube sits in a uniform electric field as shown. Which sides of the cube have the least electric flux through them?



① The front
② The back
③ The right-hand side
④ The left-hand side
⑤ The top
⑥ The bottom
⑦ There is no electric flux through any face of the cube.
⑧ Impossible to tell

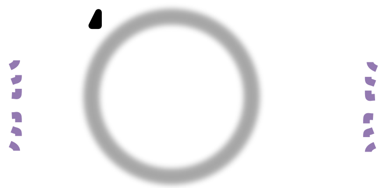
The black shapes in the picture below are closed surfaces. The colored lines are electric field lines. For which cases is the flux through the surface non-zero?



**1**

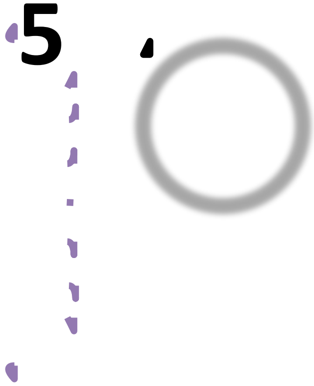
**2**

**3**

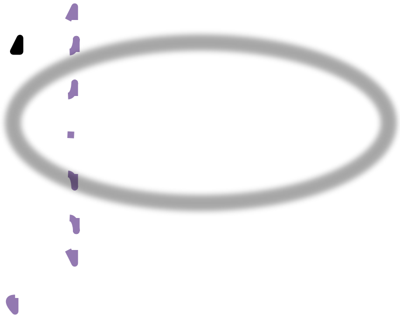


**4**

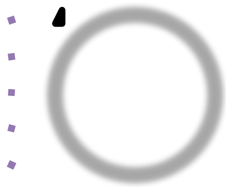
**5**



**6**



**7**

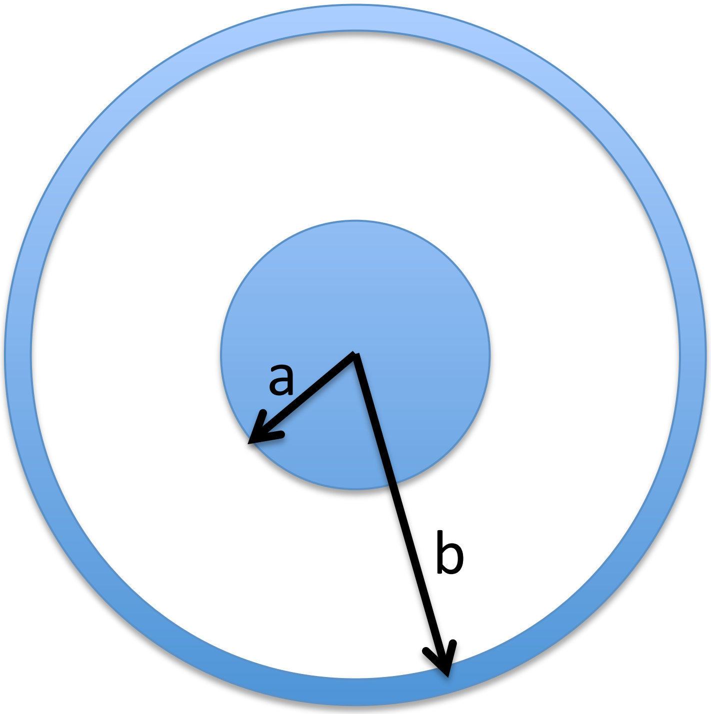


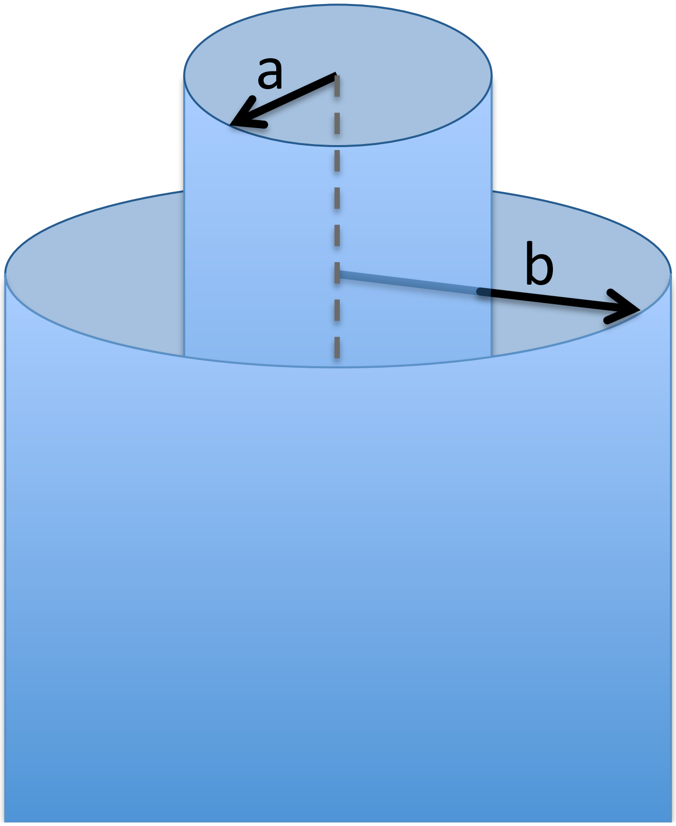
**8**

**9:** Impossible to tell.

# Coaxial Wire Worksheet

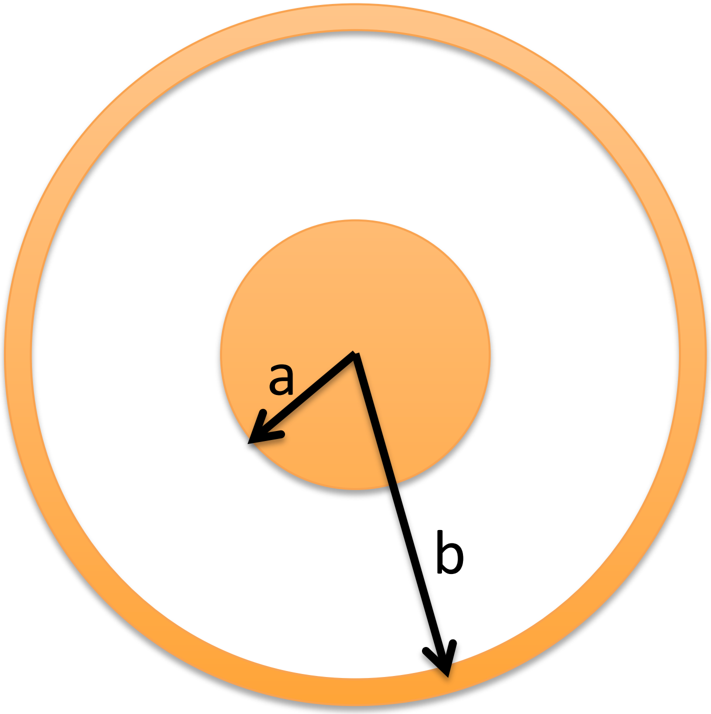
A coaxial wire consists of an inner wire (a cylinder of radius a), a layer of insulation, and an outer wire (a thin cylindrical shell at distance b from the center of the wire). Ignoring the insulator, find the electric field in *all* regions if the inner wire is charged to +q coulombs per meter, and the outer wire is charged to -q coulombs per meter.

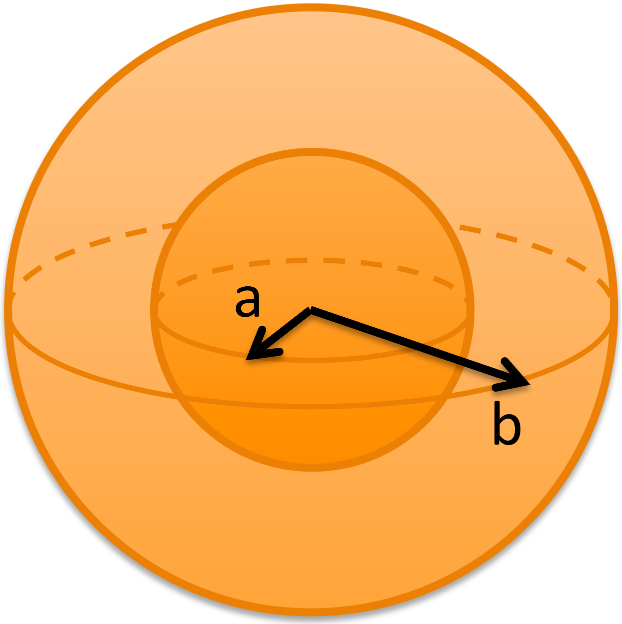




# Nested Spheres Worksheet

An insulating sphere of radius a has a total charge +q spread equally throughout it. A thin, conducting, spherical shell of radius b is placed concentrically around the inner sphere. Find the electric field in *all* regions.





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