Diffusion and Fick's Law

Probability and Statistics Series

Instructor's Guide

Table of Contents

Developed by the Teaching and Learning Laboratory at MIT
for the Singapore University of Technology and Design

Introduction

When to Use this Video

- In an introductory Probability and Statistics course after discussing moments of distributions or in a Mass Transfer course while discussing Fick's Law.
- Prior knowledge: moments of distributions and concentration gradients

Key Information

Duration: 11:56
Narrator: Prof. Sanjoy Mahajan
Materials Needed:
• paper
• pencil

Learning Objectives

After watching this video students will be able to:

- Describe the difference between regular and random walks.
- Explain the structure of Fick's law for flux.

Motivation

Diffusion is all around us. Heat travels by diffusion. In our brains, signaling molecules travel from one neuron to the next by diffusion. In short, life depends on diffusion. Fortunately, using a simple model, we can understand the essential characteristics of diffusion.

Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Perform calculations to compare the first and second moments for a molecule making a regular and random walk on a one-dimensional number line.
- Estimate the time for a neurotransmitter to diffuse across a synaptic cleft.
- Estimate the time for an oxygen molecule to diffuse from the lung to a leg muscle.

Video Highlights

This table outlines a collection of activities and important ideas from the video.

Time	Feature	Comments
0:48	Prerequisite Knowledge and Learning Objectives	
1:06	Chapter 1: Regular and random walks	
2:33	Student activity	Students are asked to find the expected value (first moment) of a molecule making a random walk on a one-dimensional number line. The molecule only moves left or right by one unit at each clock tick, with equal probability of moving in each direction.
3:41	Student activity	Students are asked to calculate the second moment.
6:21	The fundamental difference between a regular and random walk is discussed using the example of a molecule crossing the gap between two neurons.	
7:16	Chapter 2: Fick's Law	
9:22	Student activity	How long would it take a neurotransmitter molecule to diffuse across a 20 nm synaptic cleft?

Video Summary

This video uses a simple model to help students understand the fundamental difference between a regular and a random walk. The concept of a random walk is connected to diffusion and is used to explain the structure of Fick's law. Finally, Fick's Law is used to explain why wearing a coat is useful in the winter, to estimate diffusion times for biomolecules, and to understand why we need a circulatory system to efficiently transport oxygen throughout the body.

Course Materials

Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.

1. Probability practice: You flip a (fair) coin three times. List the probabilities of getting 0 heads, 1 head, 2 heads, and 3 heads. Then use the methods from the Moments of Distribution video (see STEM Concept Video collection at https://tll.mit.edu/video) to find the expected number of heads.

Post-Video Materials

1. Generalizing the argument to two dimensions: The analysis of the random walk in the video used a particle wandering on a one-dimensional number line. Test whether the fundamental conclusion about a random walk, that the expected squared distance increases by one with each time tick, holds in two dimensions (hopefully it does!).

So, imagine a particle at some random location, say (2,3). When the clock ticks, it has four equally likely moves: up, down, left, or right. What is the expected squared distance to the origin, before and after the clock tick?

2. Continuous directions: Does your conclusion from the preceding problem change if the next move could be one unit in any direction, not just along the coordinate axes, with all directions equally likely?

3. Reynolds number: The random-walk motion of particles also applies to momentum; then the diffusion constant is called the kinematic viscosity, *nu*, (it is the fluid's dynamic viscosity divided by the its density). With that idea, our analysis of diffusion will give a meaning to the most important quantity in fluid mechanics: the Reynolds number. To do so, imagine fluid flowing at speed v past a sphere of radius r. Then, in terms of r, v, and *nu*, estimate the following ratio:

time for momentum to diffuse past the sphere
----
time for fluid to flow past the sphere

Look up the formula for Reynolds number, and behold!

Additional Resources

References

The use of random processes to model molecular interactions may be unfamiliar to students depending on their background. The first two references are a text about random walks in biology and an online virtual experiment about random walks of molecules.

- Berg, H. C. (1993). Random Walks in Biology. Princeton, NJ: Princeton University Press
- Blauch, D. N. (2009). Virtual Chemistry Experiments: Kinetic Molecular Theory of Gases. Retrieved from http://www.chm.davidson.edu/vce/kineticmoleculartheory/index.html

For another approach to helping students understand Fick's first law, check out the following STEM Concept Video:

- French, J. E. (Writer), (2012). *Gradient*. Available from http://ocw.mit.edu/resources/res-tll-004-stem-concept-videos-fall-2013/videos/differential-equations/gradient/